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ORIGINAL ARTICLE

Response times seen as decompression times in Boolean concept use

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Abstract This paper reports a study of a multi-agent model of working memory (WM) in the context of Boolean concept learning. The model aims to assess the compressibility of information processed in WM. Concept complexity is described as a function of communication resources (i.e., the number of agents and the structure of communication between agents) required in WM to learn a target concept. This model has been successfully applied in measuring learning times for three-dimensional (3D) concepts (Mathy and Bradmetz in Curr Psychol Cognit 22(1):41–82, 2004). In this previous study, learning time was found to be a function of compression time. To assess the effect of decompression time, this paper presents an extended intra-conceptual study of response times for two- and 3D concepts. Response times are measured in recognition phases. The model explains why the time required to compress a sample of examples into a rule is directly linked to the time to decompress this rule when categorizing examples. Three experiments were conducted with 65, 49, and 84 undergraduate students who were given Boolean concept learning tasks in two and three dimensions (also called rule-based classification tasks). The results corroborate the metric of decompression given by the multi-agent model, especially when the model is parameterized following static serial processing of information. Also, this static serial model better fits the patterns of response times than an exemplar-based model.

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Introduction

Mathy and Bradmetz (1999, 2004) (see also Mathy, 2002) conceived a set of simple multi-agent models of Boolean concept complexity (the complete set of Boolean concepts in two and three dimensions is shown in Fig. 1). These multi-agent models aim to express the compressibility of a conceptual structure in the fewest number of decisions made by agents to know the category of all examples of a concept. This article switches from conceptual complexity (assessed by learning times or accuracy) to example complexity (assessed by response times). We present three experiments showing that one of the multi-agent models (called the static serial multi-agent model because it is similar to a strict rule-based model) provides the best predictions of response times in recognition phases, despite its inability to ideally compress information. We conclude that subjects do not compress rules in an optimal way. Rather, subjects use non-optimal rules in which information is rigidly ordered. Finally, we test against the static serial multi-agent model a simple version of the exemplar model (Nosofsky, 1986). The overall results show that the multi-agent model provides a detailed description of the processing speed underlying decision-making about category membership.

Rules, algorithms and compression

In order to know whether or not a number is even, one could divide this number by two, look at the remainder, and then note if the remainder is equal to zero. There is, however, a way of avoiding this kind of procedure that imposes a new calculation for each number: it is easier to tell if a number is even by considering only the last digit. This rule is "if the last digit is 0, 2, 4, 6, 8, then the number is even". Such a rule is a compressed calculation in that it is a simplification that does not lead to any loss of information.[1]

A second kind of compression derives from the fact that stimuli are not treated alike by a rule. Each stimulus may require a particular number of steps to be processed. This is very intuitive: to checkmate with a queen against the king is easier than with a knight and a bishop; using the Erathostene's sieve method, it is easier to see that 1951 is a prime number than to see that 2209 is not because one has to reach 44 to know that 1951 is prime rather than 47 to know that 2209 is not; it is easier to recognize a gazelle in a herd of zebras than in a herd of antelopes, and so forth. This article will deal with this latter kind of compression. We will show that the time required to recognize a stimulus depends on the number of steps that have to be followed when using a given rule.

The following points describe the general problem that we will investigate:

1. Each rule is seen as an algorithm (a function) that may produce different output values depending on the given input values. (If the rule is "*red = positive example" and if a red square is given in input, the rule will produce the output "positive example".)*
2. Following the terms of the Port Royal logicians (Arnauld & Nicolle, 1662/1996), each rule is taken to be a compressed definition of a concept. In this way, a rule (intension) is more compressed than the list of examples of a given concept (extension). (The rule "*red = positive example*" is more compressed than saying that "*red square = positive example*, *red circle = positive example*, *red diamond = positive example*, etc.")
3. Each rule may be more or less compressed, given that optimizations can be found to shorten the length of a rule. This paper assumes that all rules are compressed to the maximum for the learning system considered.
4. Given a rule, some inputs require fewer steps to produce an output. For instance, the rule "(*red* = positive) *OR* (*big blue diamonds* = positive)" would take less time to indicate that a *big red square* is positive (there is one piece of information to check) than to indicate that a *big blue diamond* is positive (there are three pieces of information to check).

To sum up, given that a rule is a compressed number of operations that produces an output, the time to produce an output is a decompression time. The appendix develops the links between advanced theories of compression in computer science and processing of information, in regard to the way that compression mirrors basic economy principles of the mind. Before developing the learning system considered here, we present in the following section the material to be learned.

Concept learning

This research focuses on the ability to successfully discover and use arbitrary classification rules, also called concept learning tasks (Bourne, 1970; Bruner, Goodnow, & Austin, 1956; Levine, 1966; Shepard, Hovland, & Jenkins, 1961). In concept learning, learners are shown a sequence of multi-dimensional stimuli and formulate a hypothesis concerning the instances that do or do not belong to a category, until they inductively reach the target concept. The three basic types of classification rules in two dimensions are presented in Fig. 1, as well as the thirteen in three dimensions. Each vertex may represent the combination of Boolean input variables leading to compound stimuli wherein shape, color and size are mixed. For instance, four figures would be generated from two binary dimensions each having two values. The two category responses are represented in Fig. 1 by black circles (positive examples) and by vertices without black circles (negative examples). A concept is thus thought of as the set of all instances that positively exemplify a classification rule.

A learning system may compress the information held by different conceptual structures into simple rules, depending on the sum of the regularities present in those structures. We develop here several multi-agent models (reducible to decision tree models) to

[1] Rule creation is the motor of conceptual progress in many domains. Before Descartes, there was a procedure for each equation, depending on the terms to the right and the left of the equal symbol. Descartes came up with a considerably more economical system of calculations by putting the terms on the left and a zero on the right. This discovery brought him up against the reticence of people to accept that "something" could be equal to "nothing". Here, we see that all scientific revolutions take time. Similarly, it took a couple of decades for Kepler to admit that planetary orbits were not circular, even though elliptical orbits actually simplified the calculus.

show how humans compress concepts using simple rules but why they do not compress them using the simplest rules. We will see that this is closely linked to the computation demanded when inducing concepts.[2]

Multi-agent models of concept learning

In studying structural biases in concept learning, one investigates the system of relations in the concept to be learned and asks how the organization of relations might affect learning processes. The concept of structure is not easy to grasp: the perception of structure is a quite different matter from the perception of shapes or other physical stimuli (Lockhead & Pomerantz, 1991). A structured system can be defined as one that contains redundancy. To assess whether humans compress the information held by conceptual structures, Feldman (2000) proposed a metric based on logical compressibility of disjunctive normal forms (DNFs) describing concepts. It was shown that conceptual difficulty reflects intrinsic logical complexity on a wide range of concepts (up to four dimensions).

Using a model proposed by Mathy and Bradmetz (1999, 2004) have evaluated Feldman's model with respect to a series of multi-agent models developed to be analogous to the functioning of working memory (WM). Multi-agent models are collective problem-solving methods. This relatively old idea, developed in psychology (Minsky, 1985; Selfridge, 1959), has been recently applied in computer science for various architectures (e.g., Ferber, 1999) and in cognitive science to model social problems (Axelrod, 1997; Richards, McKay, & Richards, 2002). A multi-agent system is made of numerous mindless agents capable of reaching goals by collaboration. Competence and knowledge is not centralized but distributed among different agents who communicate with each other.

In the model proposed here, we assume that each dimension of a concept is identified by a single agent, and that information must be exchanged until the

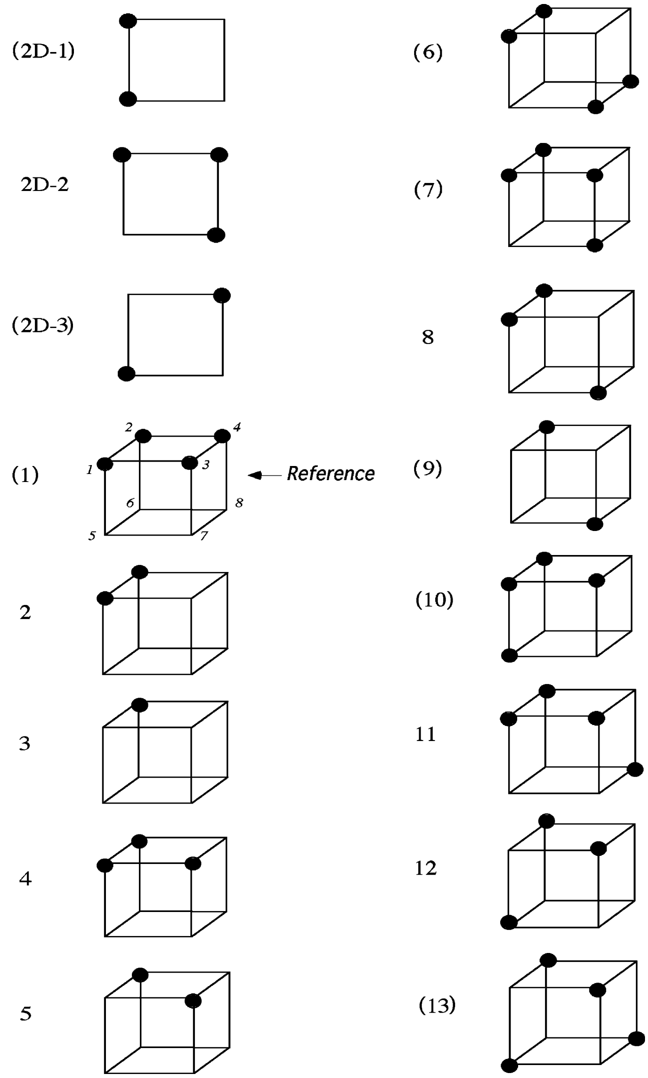


Fig. 1 Two- and three-dimensional Boolean concepts. Positive examples are indicated by *black circles*; negative examples are represented by *empty vertices*. There are only three possible concepts in two dimensions and 13 in three dimensions. The other concepts are equivalent by rotation or mirror reflection. 2D-1, 2D-2, and 2D-3 are simply arbitrary notations to distinguish the concepts in 2D from the three first ones in 3D. Concepts in parentheses are not studied in Experiments 1 and 2 because they do not lead to different patterns of response times given models

[2] In artificial intelligence, a theoretical analysis of inductive reasoning has been introduced by Gold (1967). Gold developed the notion of convergence (identification in the limit) by understanding that the most accurate hypotheses are reached faster when beginning to test the smallest ones (see also Osherson, Stob, & Weinstein, 1986, for a development of Gold theories). This principle, which consists of choosing the simplest rules is known as Occam's razor, which guarantees both fast learning and accurate generalizations (see a study of the simplicity principle in unsupervised categorization in Pothos & Chater, 2002; a study of learning based on the principle of minimum description length (MDL) in Fass & Feldman, 2002; Feldman, 2003b for a short introduction to simplicity principles in concept learning and Feldman, 2004, for a study of the statistical distribution of simplicity). Recently, computational learning theories have achieved success with the probably approximately correct (PAC) learning theory of Valiant (1984) (Anthony & Biggs, 1992; Hanson, Drastal, & Rivest, 1994a, b; Hanson, Petsche, Kearns, & Rivest; Kearns & Vazirani, 1994). This approach is a general framework (e.g., sample complexity, Vapnik–Chernovenkis dimension, etc.) for a lot of inductive learning models like neural networks or inductive logic programming (De Raedt, 1997). A second approach, which we will follow in this paper, aims to develop symbolic learning algorithms based on decision trees, and is very well suited to the nonfuzzy Boolean concepts studied here (Quinlan, 1986; see Mitchell, 1997, for a general presentation or Shavlik & Dietterich, 1990, for readings in machine learning).

stimulus presented can be identified as a positive or negative example of a given concept. Our model proposes the idea that agents are part of the WM. On the first hand, the memory span corresponds to the sum of individual pieces of information held by agents. Each piece of information is processed by an agent installed in one slot of the WM. For instance, if the memory span is seven items, it means that up to seven agents can be recruited simultaneously and/or serially to reach a decision. On the other hand, the manipulation of information between agents when they communicate is seen as constituting the executive component of the WM. In this view, the memory span and executive functions are integrated in a single model: the memory span is represented by agents and the executive function is represented by information manipulated and exchanged between agents.

The inherent advantage of this multi-agent model is that it allows us to address the issue of the nature of information processing (static serial, dynamic serial, or random) to make classification decisions. When agents communicate, do they communicate each in turn always in the same order, each in turn in a varying order, or in a random order? These different models offer several ways of compressing a given sample of examples into a logical formula. Our results indicated that the dynamic serial model leading to the most compressed formulas does not give the best fit with the experimental data (Mathy & Bradmetz, 2004). Conversely, the results confirmed that the static serial model, which imposes a fixed information-processing order, is the best model to fit the data, even if it does not lead to the maximal compression of information compared to the dynamic serial model. The aim of this paper is to verify this result in a completely different context of measurement, moving from inter-conceptual measures of learning times to intra-conceptual measures of response times. The random model, which was way behind the dynamic and static models, will not be considered in this study.

In the distributed model proposed here, agents have information about a single dimension and are unaware of what others know. If a *red triangle* is presented, the color agent knows that the stimulus is *red* and the shape agent knows that the stimulus is *triangle*. Information must be exchanged (if necessary) until the stimulus is identified as a positive or a negative example. As common knowledge is being built, speaking turns are assigned on the basis of an entropy calculation, which enable agents to compare the amounts of information they are capable of contributing (a method used in Quinlan, 1986). When an agent releases a piece of information, the agent knows how much its contribution reduces the uncertainty. For instance, if the color agent notices that all red examples are positive examples (after the sample of examples is presented at least once), the color agent is able to classify correctly these examples alone afterwards, meaning that the information gain produced by the color agent is maximal for red examples. However, if half the blue examples are positive and the other half are negative, the color agent cannot reduce the uncertainty for blue examples. In this latter case, the color agent leaves one bit of information to other agents if this agent speaks first for blue examples (in other terms, there is no information gain). To make this model more concrete, one can imagine a card game where, before each round, each agent states how much he will reduce the uncertainty by laying down a card.

The main idea that underpins the model proposed by Mathy and Bradmetz is that agents enable common knowledge (when agents communicate their pieces of information, knowledge becomes public) to be produced from distributed knowledge (agents receive only one piece of information). As in classical distributed systems in which processing is split up, each agent merely receives information from a given dimension and is blind to others. However, common knowledge made up of several pieces of information is usually necessary to solve problems. Hence, agents have to communicate as the need arises to coordinate information, and therefore progressively adapt a minimal communication structure to the problem.[3]

The communication demand can hence be considered a measure of the information complexity of a concept (see Hromkovič, 1997, for a development of communicational complexity).

Communication protocols for concept 2D-1

A communication protocol is a set of communications that agents need to follow to classify perfectly all examples of a given concept. In a communication protocol, letters represent agents and exponents represent the number of times agents are required for a single presentation of a sample of positive and negative examples. If an agent is required four times for a single presentation of a sample, the exponent will be set to

[3] This progressive adaptation recalls "in the spirit" the procedure of identification in the limit (Gold, 1967), the cascade correlation algorithm for neural networks (Fahlman & Lebiere, 1990), the RULEX model that begins with the simplest rules and adds exceptions if necessary (Nosofsky, Palmeri, & McKinley, 1994b), and the SUSTAIN model of category learning in which clusters are recruited progressively (Love, Medin, & Gureckis, 2004b).

four, meaning that this agent would be required 40 times for ten presentations of a sample (i.e., ten blocks). The communication protocols given below are the best (i.e., minimal and stable) protocols achieved by the multi-agent systems after agents are presented with several learning samples.[4] In the following examples, we explain the communication protocols produced by the static and dynamic serial multi-agent models.

For example, let us consider the two-dimensional (2D) Boolean world based on two shapes and two colors. Following the conceptual structures in Fig. 2, it is assumed that the two stimuli on the left are blue, the two on the right are red, the two top ones are triangles, and the two bottom ones are squares. The concept 2*D*-1 is modeled by a unary communication protocol X^4, because only one agent X (here the shape agent) is required four times to sort the four positive and negative examples of the concept. The shape agent only is necessary and sufficient to categorize the four stimuli because the concept separates the triangles from the squares. Both dynamic and static models lead to the same formula. Communication protocols can be reduced to decision trees in which only two leaves corresponding to the two categories are graphed. The decision tree associated with the formula X shows that, if the stimulus is a triangle, the agent X will follow the left branch and conclude that the stimulus is a triangle (because the leaf is marked with a positive symbol); in the case where the stimulus is a square, X will follow the right branch and conclude that the stimulus is a negative example of the concept (the leaf is a negative example). Because X is required four times (again, for a single presentation of the training sample), the exponent is equal to four.

Communication protocols for concept 2D-2

We now explain concept 2*D*-2 labeled $X^4[Y]^2$ for the static serial model: this concept requires a partial interaction between two agents, indicated by the brackets. In the static serial model, there are two equivalent possibilities of ordering agents. The first possibility is that the color agent speaks first. For the two red stimuli, the color agent (X) will be sufficient to conclude that the stimuli are positive examples. For the two red stimuli, the color agent will follow the left branch in the decision tree. In contrast, when speaking first for the blue ones, the color agent leaves one bit of information and the second speaker Y (the shape agent) will be necessary to complete the task. In the decision tree, when the color agent makes its information public for the blue ones, it follows the right plain branch of the tree. When the second agent (the shape agent) gives its information, the shape agent follows the left dotted line when the stimulus is triangle and the right one when it is a square. The ... [...] binary operator indicates that Y is not required all the time: the interaction between X and Y is therefore partial. Given that the second speaker is recruited only twice when presenting a single sample of examples, we can set the exponent to 2. Note that there is a second static serial order when the shape agent speaks first and the color agents speaks second: In this case, the same communication protocol and the same decision tree holds.

The formula for the dynamic serial model is more compressed: $X^4[Y]^1$. The advantage of the dynamic serial model is that agents are not constrained by a fixed order of communication. The root represents the choice to be made by the first speaking agent X, no matter who he is. For that reason, both the color and the shape agents can replace X for one given concept. For the concept 2*D*-2, the color agent is sufficient to categorize the red stimuli as positive example and the shape agent is also sufficient to categorize the triangle stimuli as positive examples (one of the two agents is randomly chosen for the *red square*). In contrast, the red circle stimulus requires two agents to be sorted because an interpretation of silence is not allowed in this model (either the color agent gives its piece of information followed by the shape agent, or the shape agent gives its piece of information followed by the color agent). The best way to structurally represent dynamic serial formulas is to see them as decision trees in which the same path could be followed by several agents. $X^4[Y]^1$ therefore means that an agent X is required for all stimuli, but also that an optional agent Y is needed to classify one of the stimuli.

There are correspondences between the multi-agent model and other models of concept learning based on rules. In terms of rules and when considering only the positive examples, the static serial model would corresponds to [IF *red* THEN positive. IF *blue* THEN [IF *triangle* THEN positive]], whereas the dynamic model would simply corresponds to [IF *red* OR *triangle* THEN positive]. It is immediately apparent that the static serial model is more cumbersome, in that it

[4] We will not present the method for obtaining communication protocols, as it is already explained in Mathy and Bradmetz (2004). The method is based on computing the information gain for each piece of information given by agents until there is no more uncertainty about the class. The knowledge of an agent is computed by the conditional entropy quantifying the remaining uncertainty about the class once the agent's knowledge is made public.

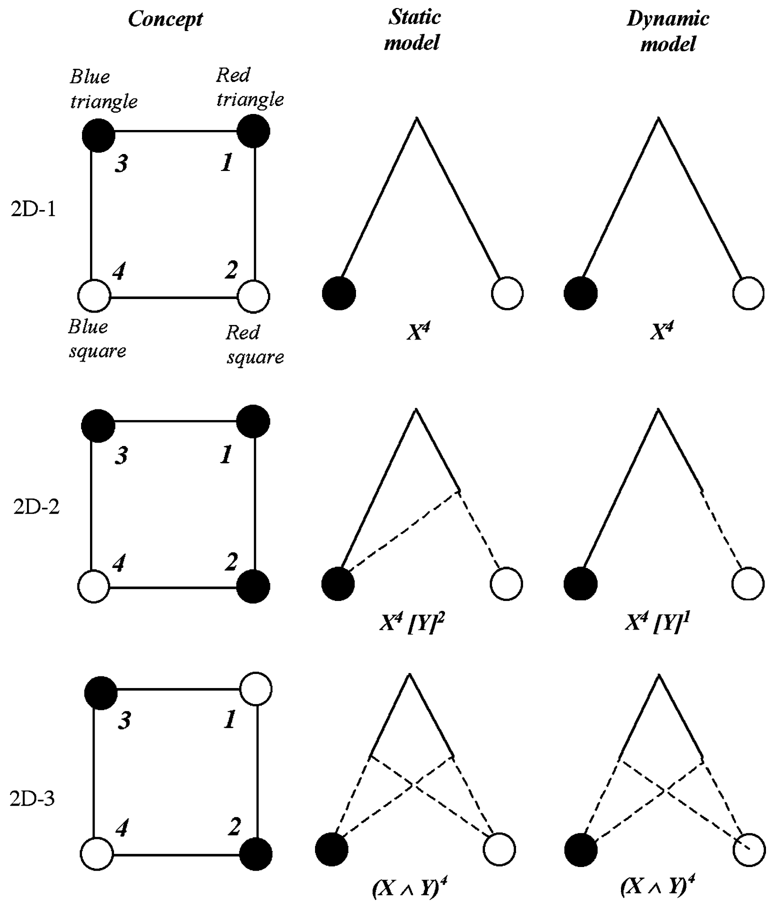


Fig. 2 The three conceptual structures in two dimensions associated with the communicational protocols required to categorize all stimuli

requires one more embedding structure. In terms of DNFs and when focusing on positive examples only (cf. Feldman, 2000), the concept 2D-2 would be represented by [*red* OR (*blue* AND *triangle*)] in the static model and by [*red* OR *triangle*] in the dynamic serial model. Note that, even if not always strictly equivalent, most formulas given by the dynamic model are similar to the ones given by both classical models of concept learning (e.g., Bourne, 1970) and recent models (Feldman, 2000). Therefore, the static serial model is the only truly novel proposition presented here.

Communication protocols for concept 2D-3

The 2*D*-3 concept could be modeled by a $X^4[Y]^4$ formula for both static and dynamic models, but in view of the fact that two agents are required for all stimuli, a new binary operator representing a complete interaction gives the following formula $X^4 \wedge Y^4$ or simply $(X \wedge Y)^4$.

Communication protocols for concepts in 3D

The same principles lead to the formulas in three dimensions. All formulas are given in Table 1.

To sum up, several key assumptions are made to describe formulas associated with each concept:

- Formulas represent the minimal inter-agent communication protocols.
- Embedded communications are reduced to a communication between a first speaker, a second speaker and so forth. Each letter *X*, *Y*, etc. stands respectively for the first and the second speaker (and so on). The number of letters (i.e., the number of agents) directly represents the number of agents in WM that are recruited in concept learning.
- The square brackets indicate that the speaker is optional and the exponent linked to the bracket indicates the number of times the nested agent has to provide a statement. The presence of square brackets also indicates a partial interaction between two agents.
- The "∧" symbol means that information provided by both agents is needed for each example in the concept. An $X \wedge Y$ formula is said to be isomorphic to a first order interaction between variables in statistics, also called complete interaction.
- When adding supplementary dimensions, inter-agent communications are either required

Table 1 Dynamic and static communication protocols triggered by 2D and 3D concepts

Concept	Dynamic	Static
(2D-1)	X^4	X^4
2D-2	$\mathbf{X^4[Y]^1}$	$\mathbf{X^4[Y]^2}$
(2D-3)	$X^4 \wedge Y^4$	$X^4 \wedge Y^4$
(1)	X^8	X^8
2	$\mathbf{X^8[Y]^2}$	$\mathbf{X^8[Y]^4}$
3	$\mathbf{X^8[Y[Z]^1]^1}$	$\mathbf{X^8[Y[Z]^2]^4}$
4	$\mathbf{X^8[Y[Z]^{1/3}]^4}$	$\mathbf{X^8[Y[Z]^2]^4}$
5	$\mathbf{X^8[Y[Z]^2]^4}$	$\mathbf{X^8[Y \wedge Z]^4}$
(6)	$X^8 \wedge Y^8$	$X^8 \wedge Y^8$
(7)	$X^8 \wedge Y^8$	$X^8 \wedge Y^8[Z]^4$
8	$\mathbf{X^8 \wedge Y^8[Z]^1}$	$\mathbf{X^8 \wedge Y^8[Z]^2}$
(9)	$X^8 \wedge Y^8[Z]^2$	$X^8 \wedge Y^8[Z]^4$
(10)	$X^8 \wedge Y^8[Z]^2$	$X^8 \wedge Y^8[Z]^4$
11	$\mathbf{X^8 \wedge Y^8[Z]^2}$	$\mathbf{X^8 \wedge Y^8[Z]^4}$
12	$\mathbf{X^8 \wedge Y^8[Z]^4}$	$\mathbf{X^8 \wedge Y^8[Z]^6}$
(13)	$X^8 \wedge Y^8 \wedge Z^8$	$X^8 \wedge Y^8 \wedge Z^8$
Sum	201.3	224

Bold concept numbers are those for which the dynamic serial model and the static serial model lead to different predictions of mean response times. The patterns of intra-conceptual response times are not automatically distinguishable when formulae are different. Concepts in parentheses are not studied in Experiments 1 and 2 because they do not lead to different patterns of response times given models

(represented by the operator $\wedge$) or optional (represented by [ ]). Communications are added in a recursive manner.

- All communication operations can also be enumerated through DNFs. For example, $X\ [Y \wedge Z\ [W]]$ can be read: $X \vee (X \wedge Y \wedge Z) \vee (X \wedge Y \wedge Z \wedge W)$. This indicates that an example of a concept requires the contribution of one or three or four nested agents communicating their information sequentially. For this reason, both the static and the dynamic model are considered as serial models because information for a given stimulus to be classified is given sequentially in both models. It can be noted again that the letters do not represent specific agents, but rather the order in which information is given. This is the main advantage of the multi-agent system. Indeed, the notation given by the multi-agent model gives a more comprehensive representation of disjunctive formulas than the extended representation based on disjunctive forms used by most logical models.

Pattern distinctions

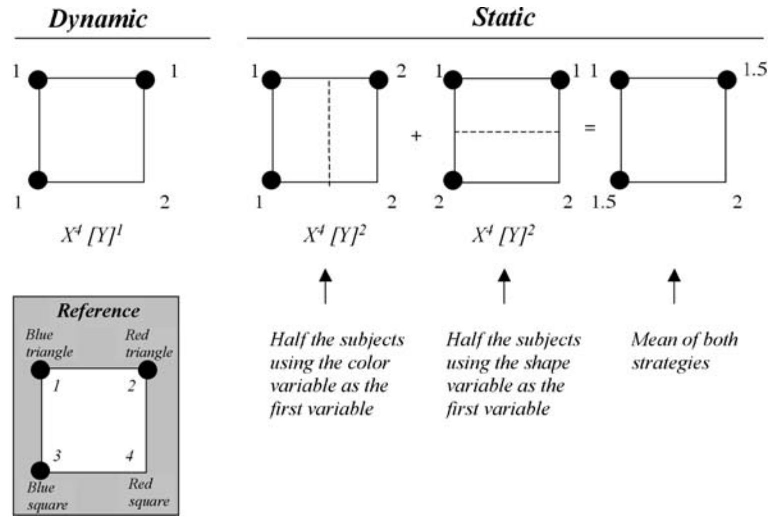
Figures 3 and 4 indicate the number of required agents for the static and the dynamic serial models, for the concept 2D-1 and 3D concepts respectively. These Figures only include concepts for which the dynamic and the static models lead to different patterns. For the static serial model, an average pattern is computed from each of the possible patterns resulting from the different orderings of agents. In Fig. 3, for example, assuming that we run the multi-agent model several times, 50% of the time two agents will be necessary to classify stimuli 2 and 4 if the color agent speaks first whereas two agents will be necessary to classify stimuli 3 and 4 if the shape agent speaks first. These two patterns lead to a mean number of 1.5 agents for stimulus 2.

The same way of computing the mean number of agents in the static serial model has been applied for all concepts in three dimensions in Fig. 4. In three dimensions, the number of possible patterns varies from one to six because there might be six different ordering of the three agents. However, we only indicate half of them to clarify the presentation (this simply limits the number of cubes drawn). For instance, instead of indicating the pattern resulting from the order shape-color-size and a second pattern resulting from the order shape-size-color, we only indicate the pattern resulting from the average of both of them. The presence of a decimal in the computation of the number of required agents always indicates that two orders have been averaged. A dotted line within the cubes indicates the separation made when the first agent gives its piece of information. For instance, the resulting pattern resulting from the average between the order shape-color-size and the order shape-size-color, the dotted line would separate objects of different shapes.

Objectives

We make the assumption that the number of pieces of information for each stimulus indicated in Figs. 3 and 4 can easily be recovered from the analysis of response times to the learned concept (in a recognition test). The first experiment was conducted to measure response times to each instance of the 2*D*-2 concept and compare them to the patterns of the theoretical number of agents in the static and the dynamic model. The second experiment aimed to measure response times of 3*D* concepts (2, 3, 4, 5, 8, 11, 12) that also lead to different patterns of the theoretical number of agents in the static and the dynamic models. The third experiment was designed to contrast the static serial model (as we find the static model to be the more accurate in the first two experiments) with the exemplar model.

Fig. 3 Intra-conceptual analysis of the number of recruited agents in the dynamic and static multi-agent models for the 2*D*-2 concept. The number of agents corresponds to the number of pieces of information required to make the decision about category membership. Therefore, the number of recruited agents also represents the response times to make the decision about category membership



Experiment 1: two-dimensional concept use

The present experiment was designed to measure intra-conceptual response times to stimuli in a previously learned concept (i.e., during a recognition phase). Our objective was to show that stimuli require different response times to be categorized, as might be predicted from the multi-agent model where the number of pieces of information to make decisions about category membership varies. The second goal was to determine which multi-agent model (dynamic versus static) is best able to describe the pattern of response time for stimuli within each concept. We chose to begin with the 2*D*-2 concept which led to different theoretical patterns of numbers of agents in the static and the dynamic models. It is important to understand that the number of pieces of information required to identify each instance of a concept in the multi-agent models is assumed to be defined once the communication protocol is established (i.e., once the concept is learned). Accordingly, the responses times were measured after a given learning criterion had been met, ensuring that the target concept had been learned and could be applied without error.

Method

Participants

Participants were 65 high school students and university undergrad volunteers.

Stimuli

It is worth mentioning that the choice of physical dimensions is quite important in testing the cumulative effect of several dimensions in WM. In this study, input variables were compound stimuli wherein shape, color, size and a frame were amalgamated. In 2*D* (see stimuli in Fig. 5), figures varied along two dimensions each having two values, leading to a sample of four figures (e.g., a red square, a blue square, a red circle and a blue circle). The colors and shapes of the different concepts were randomly chosen from a set of values (triangle, square, oval, blue, pink, red, green, circle frame, and diamond frame).

Procedure

Tasks were computer-driven. On the day of the experiment, participants completed tutorials on a computer, which instructed them in basic computer skills and the procedures required for the experimental task. Participants were explained how to classify stimuli in two locations (either a school bag or a trash can) and how to succeed with a classification (fill up all the progress bar). The stimuli were presented in a window on the left of the school bag and the trash can (see Fig. 5). Participants were required to classify stimuli as positive examples or negative examples of a concept by using the mouse to click on a school bag or on a trash can, respectively. The icon that was not chosen (the trash can or the schoolbag) disappeared so as to facilitate the association of stimuli to their respective category. Feedback was provided at the bottom of the screen, indicating if the response was right or wrong and adding a picture of a smiling or an angry man. Each correct response scored one point on a progress bar, represented by an empty box that was filled in when they gave a correct response. The number of points in the progress bar dedicated to learning was equal to twice the length of the training sample, that is

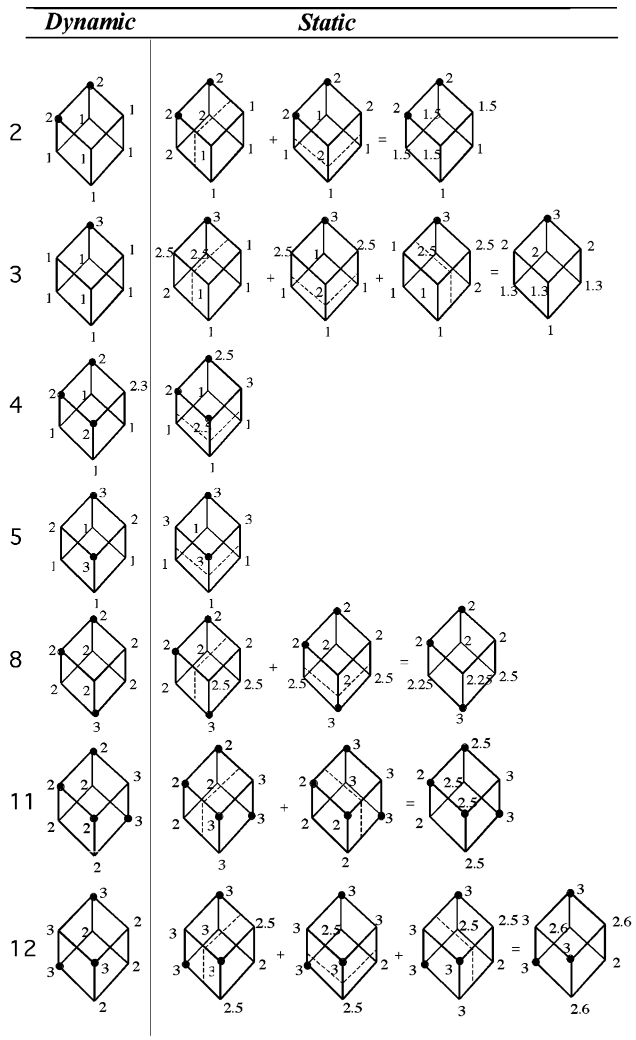


Fig. 4 Intra-conceptual analysis of the number of required agents in the dynamic and static multi-agent models for concepts 2, 3, 4, 5, 8, 11, and 12. Patterns 1, 2, and 3 of the static model are shown from left to right. The last displayed pattern is the average pattern resulting from all possible orders of agents. Because six patterns are possible (resulting from the six possible orderings of agents), they are averaged by pairs to produce a maximum display of three patterns. The nth averaged pair is represented by the nth displayed pattern

2×2^N (N = number of dimensions). Once the concept had been learned, response times were measured on a further 2×2^N points. Consequently, subjects had to correctly categorize stimuli on four consecutive blocks of 2^N stimuli. For example, participants learning a 2D concept had to fill up a progress bar of 16 points. Participants were not informed that the second half of the testing was a recognition test.

On each trial, a response dead line of eight seconds was imposed. Failure to meet the deadline cost participants three points on their progress bar. However, a wrong response resulted in loss of all points scored so far. Correct response was rewarded with a digital image (animals, fractals, etc.) when they succeeded. In Experiment 1, the stimuli varied along two binary-valued dimensions (see the four stimuli in Fig. 5). In each block of 2^N stimuli, each stimulus appeared once in a random order, and the first stimulus of each block was different from the last of the previous block. An assignment of physical dimensions was randomized for each concept and each subject. In Experiment 1, all subjects started the experiment after a short warm-up trial.

Results

Response times are summarized in Fig. 6. The response times showed a lot of variability, and they were positively skewed due to a few extreme scores corresponding to subjects who took more time to respond.

Consequently, we indicate in Table 2 median response times and base-e logarithms of response times so as to remove the effect of these extreme scores on further analysis of the data. Median response times or base-e logarithms of response times show a very good fit with the number of agents per stimulus[5] predicted by the static model. That is, the response times have higher medians for stimuli that require more pieces of information. The within-subjects analysis of variance applied on base-e logarithm response times confirms that stimuli are not categorized at the same rate [$F(3,192) = 5.16$; $P = 0.002$].

To test the agreement of data with the dynamic or static models, we computed correlations between the response times per subject and the number of agents per stimulus for each pattern of the static and the dynamic models. Then, we determined which model (dynamic vs. static) was the closest to the subject patterns (a pattern for a given subject is the set of empirical response times for a given concept). Note that in the static model, there are two theoretical patterns corresponding to the two manners of ordering the variables in the 2D-2 concept (as shown in Fig. 3). Then, we counted the number of times the static model was superior to the dynamic serial model. The results are shown in Table 3. For the 2D-2 concept, the results go against the dynamic serial model: on 50 occasions (order 1: 30; order 2: 20) out of 65 the response times are closer to the static serial model [$\chi^2(1) = 18.8$; $P < 0.001$]. Thus, the static model proved to be significantly superior to the dynamic model, with a greater correlation between the number of required agents per

[5] The numbering of stimuli used in Table 2 (ex_1, ex_2, ex_3, and ex_4) is shown in Fig. 3.

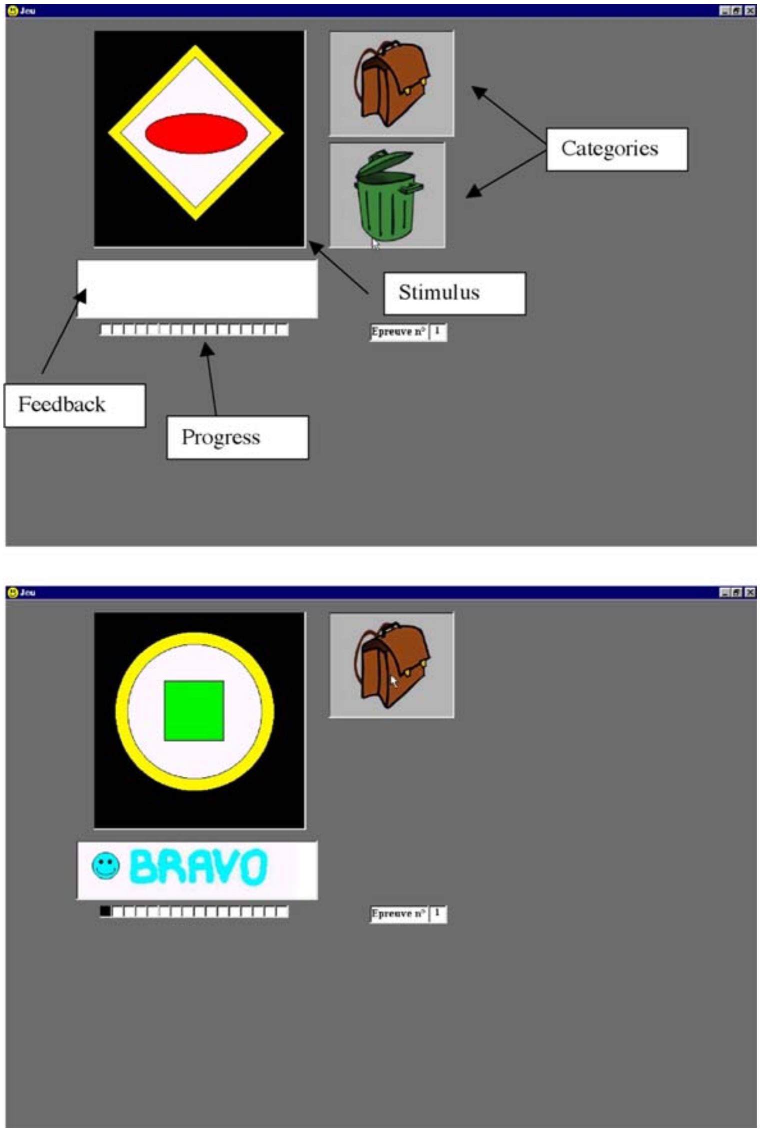


Fig. 5 Screen shot of windows in Experiments 1, 2, and 3

stimulus and the response times. This suggests modeling information processing in WM using distributed models that process information in a fixed order.

Discussion

When response times were measured in a recognition phase, our results showed that the static serial model yielded a valid measure of processing speed when categorizing stimuli of the $2D$-2 concept. In conclusion, the measure of response times sheds lights on how information is processed in WM, which uses two memory slots in a static serial way. The stimuli are not categorized at the same rate because the decompression time of the algorithm used in WM does not use the same number of pieces of information for all stimuli. To classify the positive examples, the corresponding decision rule of this static communication protocol is "if x_1 then *ex*+, if x_2 then [if y_1 then *ex*+]". This is not intuitive compared to the more compressed rule produced by the dynamic model (if x_1 then *ex*+, if y_1 then *ex*+). The rule produced by the dynamic model is equivalent to the minimal DNF. Consequently, this result casts doubt on models that use compression of DNF as a metric of conceptual complexity (cf. Feldman, 2000). This result also challenge models based on neural networks: a simple perceptron would obviously set the weights on x_1 and y_1 to sufficient values to make the output fire for x_1, y_1, or both (although it is not obvious what would predict strict parallel models in this case when dimensions are seen as stochastic variables; cf. Townsend & Wenger, 2004).

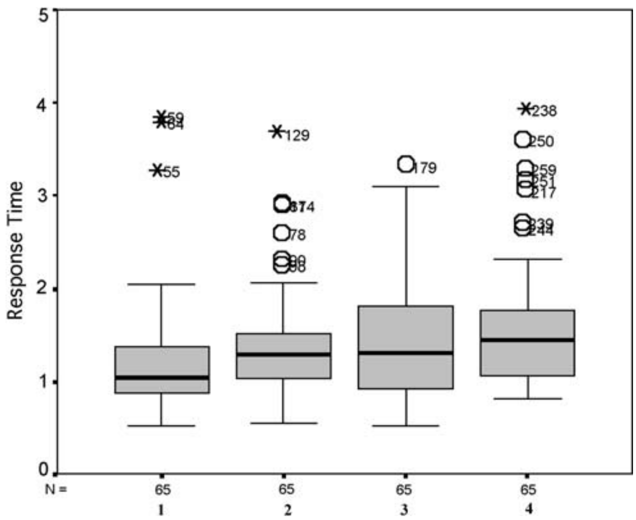


Fig. 6 Boxplots of response times for both positive and negative examples of the 2*D*-2 concept. The stimulus labels 1–4 are given in Fig. 3

Table 2 Response times for both positive and negative examples of the 2D-2 concept studied in Experiment 1

	Examples			
	1[a]	2	3	4
Mean RT	1.22	1.41	1.45	1.58
SD (RT)	0.64	1.57	0.69	0.69
Mean ln (RT)	0.11	0.27	0.27	0.38
Median RT	**1.04**	**1.29**	**1.30**	**1.45**
Static	**1**	**1.5**	**1.5**	**2**
Dynamic	1	1	1	2

RT response times, *SD* standard deviation of mean time, *ln* natural logarithm, *Static* mean number of agents per example required by the static model, *Dynamic* number of agents per example required by the dynamic model

[a] Examples 1–4 in the 2*D*-2 concept are indicated in Fig. 3. Bold lines indicate the closest patterns

Experiment 2: three-dimensional concept use

Analysis of response times in Experiment 1 clearly indicated that adults use 2D concepts by following the static serial communication complexity given by the multi-agent model. Experiment 2 aims to assess whether these findings remain valid when the target concepts are based on three dimensions.

Method

Participants

This experiment used 49 new students, from the same population as in Experiment 1.

Procedure

Using the learning program described in Experiment 1, each participant was tested on the 13 concepts in three dimensions. Tasks were undertaken in seven sessions, one session per day. The stimuli varied along three binary-valued dimensions, i.e., shape, color and frame (see stimuli in Fig. 5). The assignment of physical dimensions was randomized for each concept and each subject. The presentation order of concepts was counterbalanced to reduce the risk of carry-over effects from one concept to the next. Following the criteria described in Experiment 1, participants had to fill up a progress bar of 32 points. The response times were measured for the last 16 correct responses. Finally, they were rewarded with a digital image (animals, fractals, etc.) when they succeeded. Only then were they able to pause before learning another concept.

Results

We conducted an analysis of response times for the concepts listed in Fig. 4 because the dynamic model and the static model lead to different patterns of response times for these concepts. Boxplots of Fig. 7 show positively skewed patterns of response times similar to those observed in Experiment 1, simply indicating that some subjects took more time to respond. Dispersion of response times is analogous for all other concepts. Descriptive statistics are given in Table 4 (the results are also given in a more readable form in Fig. 8). For all concepts (except concept 8), the within-subjects analyses of variance on the log of response times show that stimuli are not categorized at the same rate [$F(7, 336) = 4.14$; $P < 0.001$, for concept 2; $F(7, 336) = 14.8$; $P < 0.001$, for concept 3; $F(7, 336) = 8.64$; $P < 0.001$, for concept 4; $F(7, 336) = 6.25$; $P < 0.001$, for concept 5; $F(7, 336) = 1.12$; *ns*, for concept 8; $F(7, 336) = 4.54$; $P < 0.001$, for concept 11; $F(7, 336) = 3.35$; $P < 0.01$, for concept 12].

We computed correlations between the median response times and the number of agents with a view to contrasting the dynamic and the static models. The results shown in Table 3 indicate that the static serial model always shows a better fit of the median response times. We also investigated which mode (dynamic versus static) was the closest to the subject patterns. With respect to the static serial model, there were several patterns corresponding to the several possible ways of ordering the variables (between two and six orderings, averaged by pairs, as shown in Fig. 4). As in Experiment 1, we counted the number of times the static serial model turned out to be superior to the

Table 3 Number of patterns (by subject) that fit either the static model or the dynamic one

	D	Concept	nDyn.	nStat.	Order 1	Order 2	Order 3	$\chi 2(1)$	$r_{\mathrm{Med.SM}}$	$r_{\mathrm{Med.DM}}$
Exp. 1	2D	*2D*-2	15	50	30	20	–	18.8***	0.985**	0.706
Exp. 2	3D	2	13	36	15	21	–	10.8***	0.744*	0.732*
Exp. 2	3D	3	12	37	16	12	9	12.8***	0.980**	0.869**
Exp. 2	3D	4	10	39	39	–	–	17.2***	0.943**	0.920**
Exp. 2	3D	5	15	34	34	–	–	07.4***	0.929**	0.730*
Exp. 2	3D	8	4	45	23	22	–	34.3***	0.106	0.083
Exp. 2	3D	11	10	39	27	12	–	17.2***	0.783*	0.538
Exp. 2	3D	12	6	43	16	11	16	27.9***	0.552	0.393

Order 1, Order 2, and Order 3 are represented in Figs. 4 and 9

D number of dimensions, *nDyn.* number of patterns by subject that fit the dynamic model, *nStat.* number of patterns by subject that fit the static model, $r_{Med.DM}$ correlation between the median response times given in Table 4 and the number of agents per example in the dynamic model, $r_{Med.SM}$ correlation between the median response times given in Table 4 and the mean number of agents per example in the static model

*Significant at the 0.05 level

**significant at the 0.01 level

***significant at the 0.001 level

dynamic serial model by computing the correlations between the mean response times per subject and the number of theoretical agents per stimulus. The results given in Table 4 show a decided difference between the dynamic and the static models: for all concepts in 3*D*, the static serial model better suited the data.

Discussion

We have made the assumption that the processing of dimensions by WM slots directly corresponds to the work of simple agents that use minimal inter-agent communication to identify and classify each example of target concepts. The multi-agent model takes into account the number of slots required per example and the number of communications used to classify each example. A distinction can be made according to whether communications are dynamic serial (when there is no order constraint between agents for the whole concept) or static serial (when a fixed ordering between agents is imposed for the whole concept). When response times were measured after the 3D concepts were learned, our results showed that the static serial model yielded a valid measure of processing speed when categorizing stimuli, as in Experiment 1. Indeed, the theoretical computation of the number of pieces of information per example (when processing is static) seems to predict patterns of response times for the 3D concepts.

When comparing the number of agents required for each example of the 3D concepts, a clear outcome is that static serial processing of information leads to less compressed communication protocol formulas than the ones given by the dynamic serial model. This indicates that suboptimal rule compressions may be privileged by human learners. One explanation could be that static serial processing in the multi-agent model leads to lower compressions of communication protocols but communication protocols are generated faster by the system (Mathy & Bradmetz, 2004). When learning concepts people would have a better performance at the end using the dynamic method, but the time required to learn the concept would be greater.[6]

Experiment 3: static serial model versus exemplar models

Links to prototype and exemplar models of categorization

The third experiment was designed to contrast the static serial model with an exemplar model. We will see that the relation between these models is peculiar for some concepts, when mean theoretical response times produced by both models are perfectly correlated. Exemplar models, as opposed to prototype models, are very well suited to nonlinearly separable concepts, like some of those in this study. These models are a generalization

[6] Let us make an analogy: when memorizing before dialing a phone number, it takes less time to dial a number after its entire memorization (let us imagine 6 s to memorize the entire number plus 3 s to dial, for a total of 9 s), than to quickly look up and memorize the numbers and dial them group by group (e.g., four groups, and 3 s per group, for a total of 12 s). Nevertheless, a lot of people choose the second solution because starting to memorize an entire number takes more time (i.e., 6 s in our example) than directly memorizing the first group and then dialing it (i.e., 3 s). Of course, this analogy should be experimentally examined.

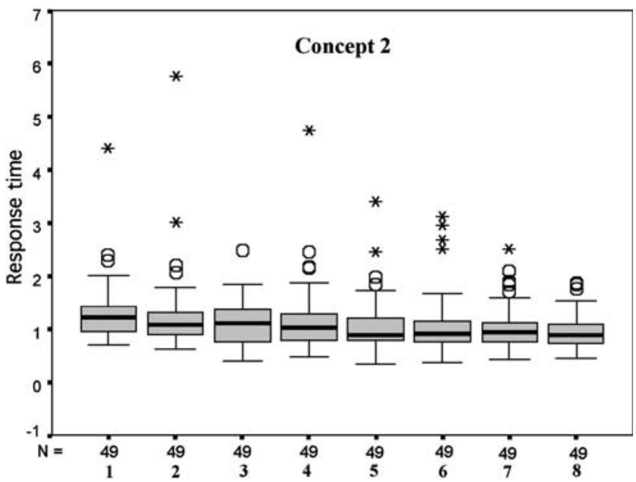


Fig. 7 Boxplots of response times for both positive and negative examples of the concept 2. The example labels 1–8 are given in Fig. 1 in concept 1

of the prototype models because they assume that the exemplar that has the highest probability of belonging to a category is the prototype. However, it is difficult to understand the role of a prototype in nonlinearly separable concepts because the prototype does not provide a good summary of the category members (Yamauchi, Love, & Markman, 2002). In exemplar models, categorization is based on the computation of similarities within a set of exemplars stored by subjects (for a review, see Hahn & Chater, 1997). According to similarity-based approaches, the more similar an item is to all known members of a category, the more likely this item will be placed in this category. Exemplar models are also called context models because exemplars form a context for computing similarities between an item and each exemplar of a category (Estes, 1994; Kruschke, 1992; Medin & Schaffer, 1978; Nosofsky, 1986; Nosofsky, Gluck, Palmeri, McKinley, & Gauthier, 1994a, b; Nosofsky, Kruschke, & McKinley, 1992).

The objective of the third experiment is to show the correspondence between the rule-based patterns of response times given by the static serial multi-agent model and the ones produced by the exemplar model. The main goal is not really to make models compete as we do not use the most complex version of the exemplar model (that would be the case if we implemented all parameters and if we used continuous weighting values), but rather to show that very similar patterns of response times are given by the static serial multi-agent

Table 4 Means and median response times of both positive and negative examples of concepts 2, 3, 4, 5, 8, 11, 12 (in Exp. 2), and concept 10 (in Exp. 3) once they are learned

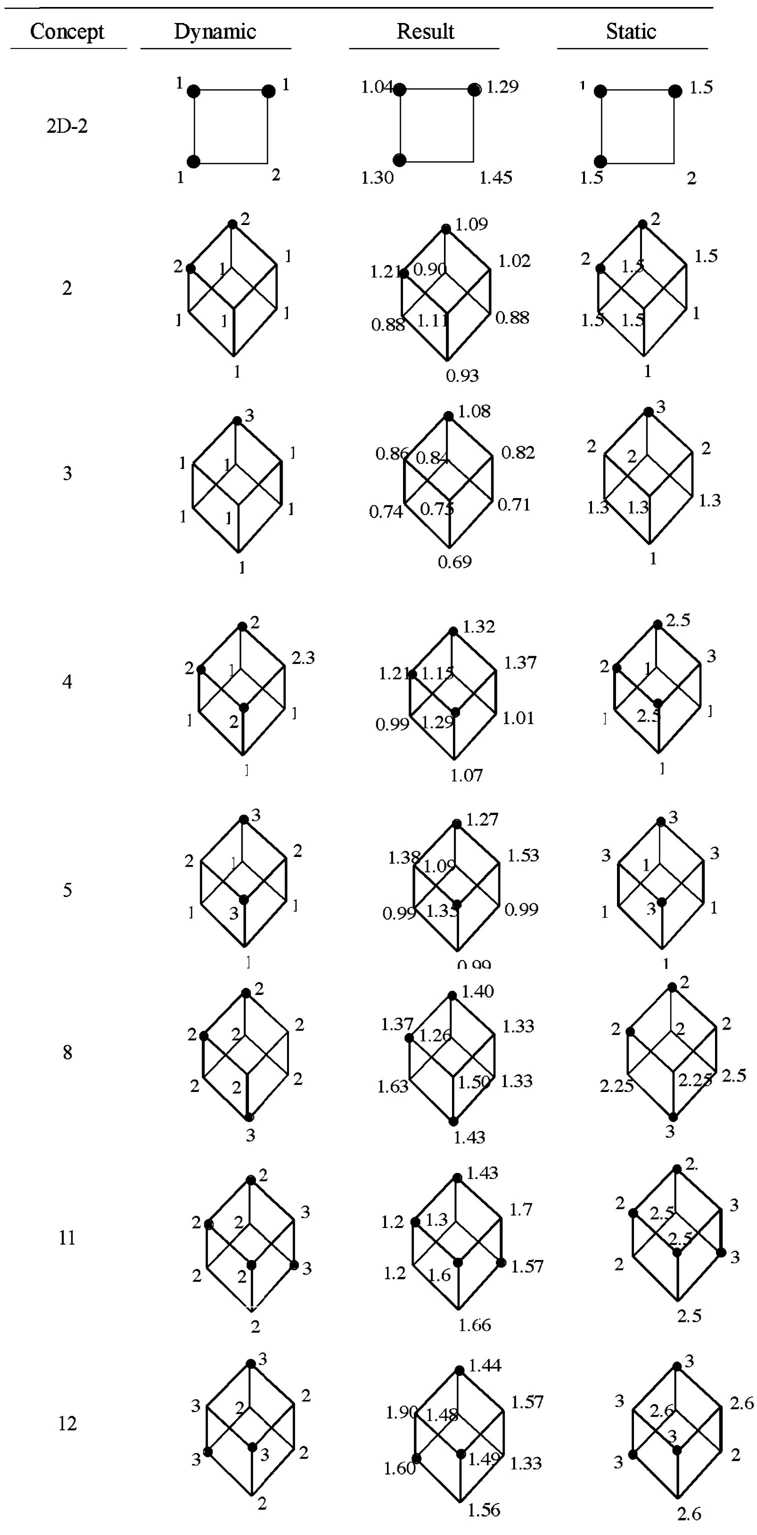
	Concept 2				Concept 3				Concept 4				Concept 5			
	M	Me	SM	DM	*M*	Me	SM	DM	*M*	Me	SM	DM	*M*	Me	SM	DM
ex_1	1.31	**1.21**	**2**	2	0.93	**0.86**	**2**	1	1.40	**1.21**	**2**	2	1.55	**1.38**	**3**	2
ex_2	1.26	**1.09**	**2**	2	1.13	**1.08**	**3**	3	1.51	**1.32**	**2.5**	2	1.49	**1.27**	**3**	3
ex_3	1.11	**1.11**	**1.5**	1	0.81	**0.75**	**1.3**	1	1.44	**1.29**	**2.5**	2	1.39	**1.35**	**3**	3
ex_4	1.16	**1.02**	**1.5**	1	0.86	**0.82**	**2**	1	1.58	**1.37**	**3**	2.3	1.59	**1.53**	**3**	2
ex_5	1.09	**0.88**	**1.5**	1	0.80	**0.74**	**1.3**	1	1.11	**0.99**	**1**	1	1.05	**0.99**	**1**	1
ex_6	1.09	**0.90**	**1.5**	1	0.91	**0.84**	**2**	1	1.23	**1.15**	**1**	1	1.19	**1.09**	**1**	1
ex_7	1.01	**0.93**	**1**	1	0.78	**0.69**	**1**	1	1.21	**1.07**	**1**	1	1.22	**0.99**	**1**	1
ex_8	0.95	**0.88**	**1**	1	0.76	**0.71**	**1.3**	1	1.16	**1.01**	**1**	1	1.25	**0.99**	**1**	1
$r_{\text{model-Med}}$			**0.74**	0.73			**0.98**	0.87			**0.94**	0.92			**0.93**	0.73

	Concept 8				Concept 11				Concept 12				Concept 10 (Exp. 3)			
	M	Me	SM	DM	*M*	Me	SM	DM	*M*	Me	SM	DM	*M*	Me	SM	EM
ex_1	1.60	**1.40**	**2**	2	1.52	**1.28**	**2**	2	2.00	**1.90**	**3**	3	1.26	**1.38**	**2**	1.22
ex_2	1.50	**1.37**	**2**	2	1.55	**1.43**	**2.5**	2	1.75	**1.44**	**3**	3	1.58	**1.68**	**2.66**	1.56
ex_3	1.54	**1.50**	**2.5**	2	1.78	**1.64**	**2.5**	2	1.69	**1.49**	**3**	3	1.51	**1.74**	**2.66**	1.56
ex_4	1.38	**1.33**	**2**	2	1.88	**1.73**	**3**	3	1.73	**1.57**	**2.6**	2	1.49	**1.64**	**2.66**	1.56
ex_5	1.63	**1.63**	**2**	2	1.45	**1.26**	**2**	2	1.80	**1.60**	**3**	3	1.46	**1.74**	**2.66**	1.56
ex_6	1.46	**1.26**	**2**	2	1.57	**1.35**	**2.5**	2	1.68	**1.48**	**2.6**	2	1.45	**1.67**	**2.66**	1.56
ex_7	1.53	**1.43**	**3**	3	1.78	**1.66**	**2.5**	2	1.77	**1.56**	**2.6**	2	1.68	**1.76**	**2.66**	1.56
ex_8	1.50	**1.33**	**2.5**	2	1.70	**1.57**	**3**	3	1.53	**1.33**	**2**	2	1.32	**1.43**	**2**	1.22
$r_{\text{model-Med}}$			**0.11**	0.08			**0.78**	0.54			**0.55**	0.25			**0.96**	0.96

Bold columns indicate the closest patterns

M mean response times, *Me* median response times, *SM* mean number of agents in the static model, *DM* number of agents in the dynamic model, *EM* theoretical response times for the exemplar model, $r_{model\text{-}Med}$ correlation between the model and the median response times

Fig. 8 Theoretical intra-conceptual analysis of the number of required agents in dynamic and static mode for the concepts 2*D*-2, 2, 3, 4, 5, 8, 11, and 12, and empirical results given as median response times per example



model without relying on similarity computations. However, when models are given equivalent chances, the static serial model turns out to fit better individual patterns of response times than the ones produced by the exemplar model.

Following Nosofsky's (1986) generalized context model of categorization (GCM), exemplars are represented in a psychological space. Distance between two stimuli *i* and *j* is given by the Minkowski metric

$$d_{ij} = \left[\sum_{a=1}^{n} \left| x_{ia} - x_{ja} \right|^{r} \right]^{1/r}, \tag{1}$$

where $r = 1$ when the distance is city-block, and where x_{ia} is the value of stimulus i along dimension a. Similarity η between two stimuli i and j is an exponentially decreasing function (called the exponential decay function) of psychological distance

$$\eta_{ij} = e^{-d_{ij}}. \tag{2}$$

This decay function is better adapted to the city-block metric (Shepard, 1987). Given the total similarity of a stimulus s to all exemplars of categories X and Y, the probability of responding with category X is given by Luce's choice rule:

$$P(X/s) = \frac{\sum_{x \in X} \eta_{sx}}{\sum_{x \in X} \eta_{sx} + \sum_{y \in Y} \eta_{sy}}. \tag{3}$$

In order to make a comparison with the static serial model, we computed the similarities among stimuli in all Boolean concepts studied in Experiments 1 and 2 using the three equations above. We used a city-block metric (adequate for separable dimensions) in a traditional multi-dimensional scaling model (Minkowski Metric), and transforming similarities in probabilities by the Luce's (1963) choice rule (cf. Chapter 10 in Lamberts, 1997). We found that probabilities of classification of exemplars are inversely related to the mean number of pieces of information for the static serial model. That is, the exemplar ex_1 in Fig. 3 has the highest probability of being classified as a positive example and is considered a prototype. Exemplars ex_2 and ex_3 are equally considered as having a medium probability of being positive examples and ex_4 has the lowest probability of being classified as a positive example. If we hypothesize as Nosofsky and Palmeri (1997) and Nosofsky and Alfonso-Reese (1999) successfully did, that the response times depend on the similarity pattern of a stimulus to the exemplars from both categories, the pattern given by the exemplar model is very similar to the one given by the mean static serial model. To summarize, when taking the inverse probabilities given by the exemplar model to measure response time, the inverse probabilities are correlated with the theoretical response times determined by the static serial model (cf. last column in Table 5).

Experiments 1 and 2 showed that the static serial model best fits the data of the present study. We will therefore consider only the static model as a comparison with the exemplar model. The major difference between the exemplar models and our static serial multi-agent model is that mean theoretical patterns of response times in the static serial model is a mixture of several static serial strategies that may be used by subjects, whereas the exemplar model computes just one pattern.

In this experiment, we compare the static serial model to GCM. Concept 10 serves as a basis for the comparison between the two models, as the patterns of theoretical response times they produce are perfectly correlated for this concept. A second advantage of this concept is that the six possible orders of variables lead to the same equivalent decision trees. These orders are shown in Fig. 9, together with the three different static serial orders that can be distinguished from them. The theoretical mean response times computed by mixing the different static orders are given in the last row of Fig. 9. This experiment was run with fixed stimuli (see top of Fig. 9) in order to precisely study the distribution of subjects' serial strategies. We also add a comparison of GCM and the static model for each of the concepts studied in Experiments 1 and 2.

Method

Participants

This experiment included 84 naive students, from the same population as in Experiments 1 and 2.

Procedure

Using the learning program described in Experiment 1, each participant was given the concept 10. The stimuli varied along three binary-valued dimensions. The assignment of values of shape, color, and filling were the same for all subjects (top of Fig. 9). This method is necessary to detect which of the six serial strategies subjects are following. Using the criteria described in Experiment 1, participants had to fill up a progress bar of 32 points. The response times were measured for the last 16 correct responses. Subjects responded using the keyboard.

Results

The mean and median response times for concept 10 are given in Table 4, next to the theoretical response times for the static serial model and the exemplar model. As in Experiments 1 and 2, the boxplots of Fig. 10 show a positively skewed pattern of response times, indicating that median response times are more representative than means. The correlations between

Table 5 Number of patterns (by subject) that fit either the exemplar model or the static serial one

	D	nS	Concept	nEx.	nStat.	Order 1	Order 2	Order 3	$\chi^2(1)$	$r_{\text{Ex.Stat}}$	$r_{\text{Med.Ex}}$	$r_{\text{Med.Stat}}$
Exp. 1	2D	65	2*D*-2	19	46	27	19	–	11.2***	0.925**	0.847**	0.985**
Exp. 2	3D	49	2	13	36	14	22	–	10.8***	0.925**	0.770*	0.744*
Exp. 2	3D	49	3	13	36	15	12	09	10.8***	0.859**	0.918**	0.980**
Exp. 2	3D	49	4	08	41	41	–	–	22.2***	0.859**	0.836**	0.943**
Exp. 2	3D	49	5	12	37	37	–	–	12.8***	0.744*	0.524	0.929**
Exp. 2	3D	49	8	13	36	17	19	–	10.8***	0.603	0.259	0.106
Exp. 2	3D	49	11	11	38	26	12	–	14.9***	0.945**	0.682	0.783*
Exp. 2	3D	49	12	05	44	16	10	18	31.0***	0.883**	0.273	0.552
Exp. 3	3D	84	10	15	69	24	19	26	34.7***	1***	0.956**	0.956**

The $\chi^2(1)$ is meant to compare columns nEx. and nStat

D number of dimensions, *ns* number of subjects, *nEx*. number of patterns by subject that fit the exemplar model, *nStat*. number of patterns by subject that fit the static serial model; Order 1, Order 2, and Order 3 are represented in Figs. 4 and 10, $r_{Ex.Stat}$ correlation between the theoretical response times in the exemplar model and those in the static serial model, $r_{Med.Ex}$ correlation between the medians given in Table 4 and the theoretical response times in the exemplar model, $r_{Med.Stat}$ correlation between the medians given in Table 4 and the theoretical response times in the static serial model

*Significant at the 0.05 level

**significant at the 0.01 level

***significant at the 0.001 level

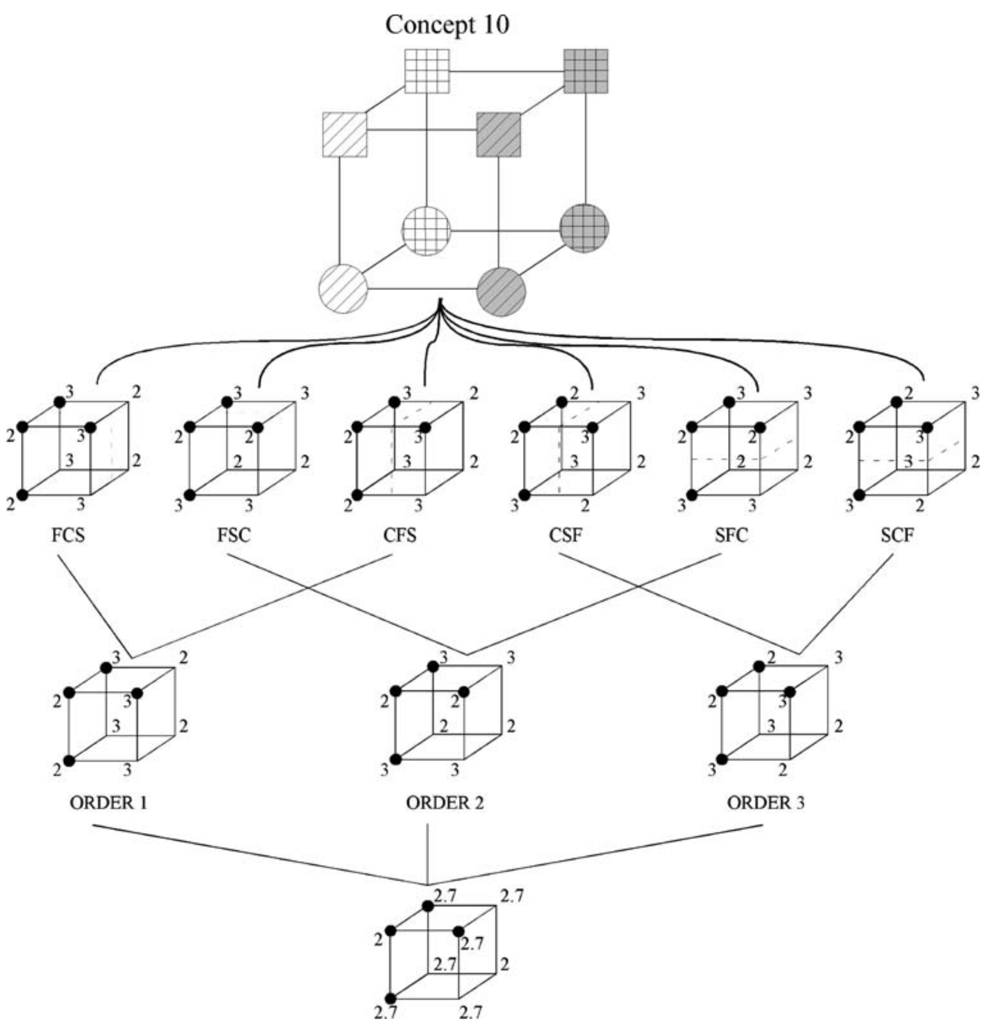


Fig. 9 Modeling of concept 10 by the static serial model. *F* filling, *C* color, *S* shape, *FCS* means that the order of decisions is *filling, color*, and *shape*

the median response times and predictions from both models are shown in Table 5. A subject-by-subject analysis of results is necessary since mean response times across subjects confirm both models. Table 5 shows that when looking at individual patterns, the static serial model explains the results better in 69 out of 84 subjects than the exemplar model [$\chi^2(1) = 34.7$; $P < 0.001$]. The distribution of strategies among the three indistinguishable orders (order 1: 24; order 2: 19; order 3: 26) is uniform [$\chi^2(2) = 1.1$; *ns*], meaning that subjects randomly chose the order of variables in their static serial decisions. This result indicates that mean response times are better explained as a mixture of static serial decisions than by patterns given by GCM.

We applied the same method to all concepts studied in Experiments 1 and 2. Table 5 displays the distribution of strategies among the three possible orders given in Fig. 4 (from left to right), but the distribution is less informative here than in Experiment 3 because dimensions were randomly chosen in these experiments. The correlations between median times given in Table 4 and theoretical response times are more often higher for the static model than for the exemplar model. The subject-by-subject analysis better shows the superiority of the static serial model. For instance, regarding the 2*D*-2 concept in Experiment 1, we tested which of the three theoretical patterns (two from the static serial model and one from GCM) had the best fit to subject response times. It turns out that subject performance is closer to one of the two static serial patterns 46 times (order 1: 27; order 2: 19) out of 65 [$\chi^2(1) = 11.2$; $P < 0.001$]. The superiority of the static serial model is also corroborated for all concepts studied in Experiment 2.

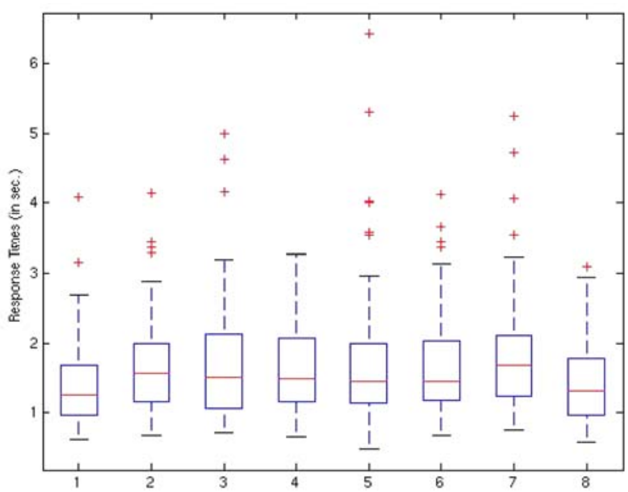


Fig. 10 Boxplots of response times of both positive and negative examples of the concept 10. The example labels are given in Fig. 1

In Table 5, the evaluation of models may be biased in favor of the static serial model because the static serial model predicts more patterns of RT than the exemplar model. For instance, for concept 3, one could argue that results are simply distributed randomly among the theoretical patterns given by both models (respectively, 13, 15, 12, and 9). Summing the number of subjects fitting one of the patterns of the static serial model (15 + 12 + 9 = 36) might give a higher a priori chance of gaining a large number of cases in favor of the static serial model. To avoid this problem, we can increase the power of the exemplar model by adding a parameter *w* (Nosofsky, 1986) called the selective attention weight on dimensions *a*:

$$d_{ij} = \left[\sum_{a=1}^{n} w_a |x_{ia} - x_{ja}|^r \right]^{1/r}. \tag{4}$$

The selective attention weight assumes that dimensions can be differentially attended to in specific contexts. This is implemented in stretching or shrinking dimensions depending on whether or not they are attended to, which causes changes when computing similarities (cf. Lamberts, 1997, chapter 10, for a detailed explanation).

Instead of taking continuous values for the attention parameter (which would give too much power to the exemplar model), we chose nine different combinations of weight in two dimensions and ten combinations in three dimensions. In two dimensions, the weights are respectively [.5 .5], [.1 .9], [.2 .8], [.3 .7], [.4 .6], [.9 .1], [.8 .2], [.7 .3], and [.6 .4]. In three dimensions, we took different basic combinations of values: One in which all dimensions are equally attended; a second in which the first dimension is more attended than the second, and the second more attended than the third; a third combination in which one dimension is more attended than the two remaining ones (which are equally attended). By taking the different permutations on the three dimensions, the ten resulting weight matrices are respectively [.33 .33 .33], [.1 .3 .6], [.1 .6 .3], [.3 .1 .6], [.3 .6 .1], [.6 .1 .3], [.6 .3 .1], [.6 .2 .2], [.2 .6 .2], and [.2 .2 .6]. Consequently, there are now a priori more chances for the data to fit the exemplar model because it produces now nine (in 2D) or ten (in 3D) theoretical patterns, whereas the static serial model produces between 1 and 6 patterns.

We computed again how many times the observed patterns fitted one of the theoretical patterns given by both models. The results are given in Table 6 that shows the frequencies of the observed patterns for each of the theoretical patterns. For instance, for concept 2D-2, 26 and 19 subjects respectively fitted the first and the second theoretical pattern given by the static serial model, whereas only 1, 3, 2, 1, 5, 3, 5 subjects, respectively, fitted the exemplar model with the weight matrices number 1, 2, 3, 4, 5, 8, and 9. To match previous tables, we summed the results by models and computed a simple χ^2. The results are seven times out of nine in favor of the static serial multi-agent model. For concept number 4, the result is not significant and for concept 10, it is significantly in favor of the exemplar model. Let us study in detail concept 4 for which the result is not significant: 31 subjects fit the average pattern [2 2.5 2.5 3 1 1 1 1] given in Fig. 4, for examples numbered [1 2 3 4 5 6 7 8]. We remind the reader that the first order is itself a computed average of two static serial patterns. For concept 4, these two patterns are [2 2 3 3 1 1 1 1] and [2 3 2 3 1 1 1 1]. There are respectively 11 and 20 subjects who fitted these patterns. For this concept, there are therefore more subjects fitting one of the patterns given by the multi agent system than any of the patterns given by the exemplar model [i.e., 20 for the static serial model against 6, the maximum frequency observed for the exemplar model; $\chi^2(1) = 7.5$, $P = 0.006$].

General discussion

Summary

Several parameterizations of a multi-agent model of WM have been conceived by Mathy and Bradmetz (2004) in order to account for conceptual complexity and to compete with logical formalizations (Feldman, 2000, 2003a). This model can be readily related to the WM functions described in earlier research. Communications correspond to the operations controlled by the executive function and the number of agents required simply corresponds to the storage capacity. Conceptual complexity is measured by the minimal communication protocol that agents use to categorize stimuli. Communication protocols are simpler to read than the formulae produced by logical formalization, as the necessary dimensions are represented only once. In our model, the communication protocol $X \wedge Y \wedge Z$ is much more understandable than its equivalent reduced DNF $x(y'z \vee yz') \vee x'(y'z' \vee yz)$. Communication protocols are also isomorphic to ordered decision trees. Contrary to other hypothesis-testing models (Nosofsky, Palmeri, & McKinley, 1994b), we presume that there is no fundamental distinction between rules and exceptions: they may simply be differentiated by the length of branches.

The static and the dynamic parameterizations already provided better predictions of inter-conceptual learning times (Mathy & Bradmetz, 2004) than logical formalizations. A second finding was that the static serial model is more accurate than the dynamic serial model. The present paper aimed at testing the static and dynamic models when predicting intra-conceptual response times. The goal was to map the complexity of learning a rule (i.e., compressing a sample of examples

Table 6 Number of patterns (by subject) that fit either one of the patterns given by the exemplar model (when the selective attention parameter is added) or the static serial one

Concept	nEx.	nStat.	Order 1	Order 2	Order 3	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	$\chi^2(1)$
2*D*-2	45	20	26	19	–	1	3	2	1	5	0	0	3	5	–	9.6**
2	34	15	13	21	–	0	2	3	2	2	4	1	0	0	1	7.4**
3	34	15	15	12	09	0	1	2	1	2	3	4	0	0	0	7.4**
4	31	18	31	–	–	0	0	4	4	6	2	1	0	1	0	3.5NS
5	37	12	37	–	–	0	2	4	0	1	0	3	2	0	0	12.8**
8	32	17	15	17	–	0	7	0	5	1	2	1	1	0	0	4.6*
11	34	15	24	10	–	0	1	1	2	7	2	1	1	0	0	7.4**
12	36	13	12	7	17	0	1	0	2	3	2	4	0	0	1	10.8**
10	26	58	7	10	9	9	9	7	7	7	5	8	2	0	8	12.2**

The $\chi^2(1)$ is meant to compare columns nEx. and nStat

nEx. number of patterns by subject that fit the exemplar model, *nStat.* number of patterns by subject that fit the static serial model; Order 1, Order 2, and Order 3 are represented in Figs. 4 and 10; For concept 2*D*-2, W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, and W_9 represent, respectively, the selective attention weights on dimensions 1 and 2, given in the format [Weight_on_dimension_1 Weight_on_dimension_2]: [0.5 0.5], [0.1 0.9], [0.2 0.8], [0.3 0.7], [0.4 0.6], [0.9 0.1], [0.8 0.2], [0.7 0.3], and [0.6 0.4]; for 3D concepts, W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9 and W_{10} represent, respectively, the selective attention weights on dimensions 1, 2 and 3, given in the format [Weight_on_dimension_1 Weight_on_dimension_2 Weight_on_dimension_3]: [0.33 0.33 0.33], [0.1 0.3 0.6], [0.1 0.6 0.3], [0.3 0.1 0.6], [0.3 0.6 0.1], [0.6 0.1 0.3], [0.6 0.3 0.1], [0.6 0.2 0.2], [0.2 0.6 0.2], and [0.2 0.2 0.6]

*Significant at the 0.05 level

**significant at the 0.01 level

***significant at the 0.001 level

given in extension into a shorter rule) to its decompression time (i.e., recovering the class of an example by applying the rule). The multi-agent models give a thorough description of intra-conceptual complexity in a recognition phase, by explaining why some stimuli are more difficult to categorize. This study showed that the static model provided better predictions of intra-conceptual response times in a recognition phase than the dynamic model.

Limitations

Use of the mouse in Experiments 1 and 2 may have introduced some noise into the response times. Use of the mouse was intentionally applied because it matched parallel research involving children. This procedure was chosen to prevent subjects from making errors of classification by pointing to the classes using a mouse (Mathy, 2002). There might be issues in measuring response times with a mouse as it is certainly a bit slower than going from one key to another. Still, the static and the dynamic models are discriminated in this study for all concepts, and the static serial model is systematically the best at fitting the data. Moreover, Experiment 3 which used keys for category responses also corroborated the static serial model.

Prediction of response times

A relevant comparison for the current work is related to neural networks applied to categorization. Unfortunately, those models (e.g., Nosofsky et al., 1994a, b) are unable to predict processing speed when categorizing stimuli. Once a neural network has set the connections between neurons, the time to produce outputs (i.e., the categories) is the same for all inputs (the stimuli), because all stimuli are categorized by the same set of connection weights. The measure of response times is also missing from the major studies that have been conducted on Boolean concepts (cf., Feldman, 2000, 2003a)

The multi-agent models we tested are able to indicate the number of minimal pieces of information required to categorize each example of a concept. We hypothesized that a stimulus requiring more pieces of information to be categorized (i.e., representing a longer path in a decision tree) would correspond to higher response times in the application phase of an already-learned concept (i.e., in a recognition phase). The second hypothesis was that the static serial model that best fitted the data in Mathy and Bradmetz (2004) would also be valid in the present experiments because the time required to induce and compress a rule (studied by Mathy and Bradmetz) is directly linked to the time needed to decompress it (studied in this article).

The results in our three experiments showed that information processing in WM is performed serially and in a static way. The static serial model better fitted the data in the first and the second experiment. These results corroborate the hypothesis that the complexity of a rule can be studied through its decompression time and confirm the better fit of the static serial model found by Mathy and Bradmetz.

The results conflict with the model of Feldman (2000, 2003a) which uses an implicit dynamic algorithm to compute the minimal Boolean formulae (although the compression algorithms are slightly different). The results also conflict with neural network models, as shown in the discussion of Experiment 1. The dynamic model allows flexible decisions as the ordering of agents can vary from one example to another. The trade-off is that more computations are necessary to compute the best ordering for each example of a given concept. The static model merely aims at producing the best ordering of agents for the whole sample of examples of a given concept. It uses simple entropy computation to determine the amount of information left by an agent. The model finds the smallest decision tree in which each level corresponds to the pieces of information given by a particular agent, meaning that the ordering of agents must be fixed when categorizing all examples of a given concept. Even though most researchers would be reluctant to return to old models in artificial intelligence based on simple decision trees (e.g., Hunt, Marin, & Stone, 1966), our results show that the static serial model corresponding to a simple decision tree model better fits the experimental results.

The problem of the time of access to categories has also recently been investigated by Gosselin and Schyns (2001) in taxonomies: the SLIP model is able to predict the time of access to categories by implementing strategies that are similar to the ones used in the 20-question game[7]

These strategies correspond directly to the computation of entropy in base 2 used in our static serial model (for instance, guessing a card of a deck of 32 requires five binary questions). Questions in the 20-question game have to be well-chosen and well-ordered to guess as quickly as possible the nature of an object (Richards & Bobick, 1988; Siegler, 1977). The same strategy drives our multi-agent model during the identification process. That is why each communication

[7] One of the two players chooses a word and the other must guess it after having asked as few yes–no questions as possible.

protocol in our multi-agent model can be seen as a tree in which each branch corresponds to a response to a binary question.

Links to prototype and exemplar models of categorization

The results are also relevant to prototype and exemplar models of categorization. Prototype theories assume that classification decisions are based on comparisons between stimuli and an abstract prototype usually defined as the central tendency of the category distribution (for an overview, see Osherson & Smith, 1981; Rosch & Mervis, 1975; Smith & Medin, 1981). The relevance of response times is well known in research based on prototype theory because the prototype is more quickly assigned to its category than other examples (for instance, see Rips, Shoben, & Smith, 1973; Rosch, 1973), but few other specific hypotheses on response times can be found in the literature, except the RT-distance hypothesis, according to which reaction times decreases with the distance in psychological space from the stimulus to the decision bound that separates categories (Ashby, Boyton, & Lee, 1994). However, decision bound models seem very inadequate when dealing with some highly non-linearly separable dimensions in Boolean concepts used in this study.

In general, prototype theories are distinguished from exemplar theories because the similarities are only computed in comparison to the prototype instead of being compared to each exemplar of the category. Some researchers (e.g., Myung, 1994) regard exemplar models as unreasonable due to the sum of computation required to compute similarities, while others find them to be very parsimonious (see the interesting study of exemplar models in avian cognition in Huber, 2001). The exemplar-based random walk model (EBRW) has also been used to account for response times in various categorization tasks by predicting that response times depend on the similarities of a stimulus to the exemplars of categories (Nosofsky & Palmeri, 1997), but the model is most likely to operate in domains involving integral dimensions. This model also involves massive similarity computations performed over these stored exemplars. The same observation can be made about another exemplar model, EGCM-RT (the extended generalized context model for reaction times), except this model provides an accurate account of categorization response times for integral-dimension stimuli and for separable-dimension stimuli (Lamberts, 2000).

The simple comparison made here between exemplar models and our multi-agent model here needs further consideration. Our use of discrete values for the attention parameter instead of taking continuous values amounts to weakening the power of exemplar models. Using continuous values in the exemplar model would have increased its general fit to the data. We used discrete values for the attention parameter in order to equal a priori chances of fitting the data in the concurrent models. We believe that a study of a possible mimicry between models would certainly be worth considering in a future study. In this regard, tools available in model selection (Roberts & Pashler, 2000; Zucchini, 2000) would certainly help decide which model is the best, depending on the number of parameters included in the models.

Contrary to previous research corroborating models through learning times and response accuracy (e.g., Love, Medin, & Gureckis, 2004a, b; Nosofsky, Gluck, Palmeri, McKinley, & Gauthier, 1994a; Shin & Nosofsky, 1992), our study showed that worthwhile research can benefit from the measure of response times using an explanation based on rule decompression. Our results also cast some doubt on research that confirmed prototype or exemplar theories by computing patterns of mean reaction times for group of subjects. Nosofsky, Palmeri, and McKinley (1994b, p. 54) also indicated that good fits of exemplar models may result from averaging over the responses of different subjects. Our results show an interesting link between the static serial model and exemplar theories. The static serial multi-agent model provides a detailed description of the cognitive processes underlying decision making about category membership. The model describes how dimensions are ordered serially to induce the minimal rule without relying on similarity as an explanatory principle. This is the major contrast between the multi-agent model investigated here and exemplar/prototype theories, because the complexity of computation of similarities is the most criticized part of exemplar/prototype models.

We find in our data a very good correlation between the static serial model and GCM for mean response times of stimulus classification. However, the three experiments (especially the third one) show that mean response times reflect a mixture of static serial decisions and not the GCM patterns.

The static versus dynamic issue

An advantage of the multi-agent models (over the most recent description of the logical complexity of Boolean concepts, e.g., Feldman, 2000) is that they allow one to address the issue of the nature of information processing (static or dynamic) in computing disjunctive formulas. The models in the present study offer several ways of compressing a given sample space by a logical rule, depending on whether the rule is computed in a static or a dynamic way. To put it simply, the static model constrains agents to communicate using the same order for all examples of a given concept whereas the dynamic model allows agents to be ordered differently from one example to another. The goal of both models is to use the minimal amount of information to classify the list of examples of a given concept. To reach this goal, the dynamic model can vary the ordering of agents from one example to another whereas the static model cannot.[8]

The dynamic model leads to the most highly compressed formulas. However, the results showed that the static model, which imposes a fixed information-processing order, best fits the data, even if it does not lead to the maximal compression of information in a rule. So why do the less compressed rules inherent in static serial processing prevail over those in dynamic processing? Mathy and Bradmetz (2004) invoke the use of constant patterns in natural language to explain why the static model prevails. Ashby, Alfonso-Reese, Turken, and Waldron (1998) support this idea by a neuropsychological theory of categorization that assumes that people have a verbal system based on explicit reasoning and a nonverbal implicit system that uses procedural learning.

Secondly, Mathy and Bradmetz (2004) showed that the sum of computations in the static serial model is very economical compared with the dynamic one. The serial functioning is equivalent to the resulting computation of an ordered binary decision diagram that orders the most informative variables first in a decision tree (OBDD, see Huth & Ryan, 2000; Bryant, 1986) with pruning carried out by computing entropy. This method avoids the combinatorial explosion coming from the comparison of the set of possible trees (for all possible ordering of variables) before obtaining the smallest path for a given stimulus. In concurrent models, the dynamic model is either implicit (Feldman, 2000) or obvious in models based on neural networks (Gluck & Bower, 1988a, b; Nosofsky et al., 1994a, b). Even in the early models (Bourne, 1970; Bruner et al., 1956; Hovland, 1966; Levine, 1966; Shepard et al., 1961), a disjunctive class in a Boolean world is always modeled as ($blue \vee triangle = positive$, for the concept of Fig. 3), whereas the serial model would describe this disjunctive rule as ($blue \vee (red \wedge triangle) = positive$) because the shape dimension cannot be processed as long as the color dimension is not. To our knowledge, no model fits the properties given by our static serial multi-agent model. This is quite surprising because the static serial model corresponds to monothetic decision trees which are most often used in artificial intelligence whereas the dynamic multi-agent model, very similar to rule-based models, corresponds to polythetic trees (in which multiple attribute values may label each tree branch) which are almost never considered in artificial intelligence for complexity reasons (see Duda, Hart, & Stork, 2001). The reason why most psychological models are not worried about polythetic decisions is certainly because psychological experiments do not use a lot of dimensions.

[8] There is an analogy with variable typing in computer science. Static-typed variables are defined at compile-time and remain unchanged throughout program execution, whereas dynamic variables are defined at run-time.

Conclusion and extensions

To conclude, we have considered throughout this paper a model of conceptual complexity based on compression of information, inspired by Kolmogorov complexity and logical depth. We found that measuring compression based on the constraints of WM is more accurate than measuring complexity based on minimal formulae in propositional logic. We refined the description of WM capacity, usually estimated by a cardinal metric, by studying communication protocols between memory slots. Communications are used to induce the minimal decision tree corresponding to a concept. We showed that decisions depend on whether information is processed in a static or a dynamic way in WM. The question raised is whether the order in which information is used in WM is constant or variable. To the best of our knowledge, this distinction has never been made before (except, again, in artificial intelligence where monothetic trees are distinguished from polythetic trees). Static and dynamic models lead to different patterns of response times when classifying stimuli. The static serial model proved to be the more accurate, suggesting that pieces of information are processed in a fixed order in WM. The last experiment also showed that response times produced by the exemplar model could be explained as a mixture of different static serial strategies. Finally, a better description of information stacking should be developed to explain details about static serial processing of information. To go further, the model could be applied to study the nonindependence of stimulus properties found in human concept learning (Love & Markman, 2003), because the model is well suited to the hierarchical processing of dimensions.

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Appendix: Compressibility and complexity

A unifying principle across many areas of cognitive science is that much of induction processing concerns compression (Chater & Vitányi, 2003) and this principle is followed in many applications (Li & Vitányi, 1997). Let's see how the notion of compression can be inserted into a general theory called algorithmic complexity. Imagine N people represented in a diagram simply as dots and each two-way communication link as a line connecting two dots. The resulting diagram could be specified by the complexity of a pattern of connection. Everyone will agree that a pattern with a lot of connections is complex, but also that having all dots connected is just as simple as having no dots connected (Gell-Mann, 1994). This reasoning suggests that at least one way of defining the complexity of a system is to make use of the length of its description, since the phrase "all dots connected" is of about the same length as "no dots connected". Computer scientists (e.g., Chaitin, 1974, 1987, 1990; Delahaye, 1993, 1994) consider a particular object described by a string of symbols and ask what programs will cause the computer to print out that string and then stop computing. The first and still classic measure of complexity that was introduced by Kolmogorov is roughly the shortest computer program capable of generating a given string (Kolmogorov, 1965). The length of the shortest program is the algorithmic complexity of the string or "Kolmogorov complexity". It corresponds to the difficulty of compression of a representation. Some strings of a given length are incompressible. In other words, the length of the shortest program that will produce one of these strings is one that says PRINT followed by the string itself. Such a string has a maximum Kolmogorov complexity in relation to its length, given that there is no algorithm that will simplify its description. It is called a random string precisely because it contains no regularity that enables it to be compressed.

Kolmogorov complexity is a measure of randomness, but randomness is not what is usually meant by complexity. In fact, it is just the nonrandom aspects of an object that contributes to its effective complexity (e.g., its structure), which can be characterized as the description of the regularities of that object. Bennett complexity shadows this type of complexity linked to the fact that an object can be highly structured, but still difficult to compute. The inadequacy of Kolmogorov complexity is striking when considering that the complexity of a string can be very high in view of the computation it needs even if the program is very short. For instance, the string of the first hundred million digits of π has a small Kolmogorov complexity, but the time needed for the program to produce the digits is high. A fractal can also be represented by a short algorithm, but it takes a long time to compute. This computational content is called logical depth, organized complexity, or Bennett complexity (Bennett, 1986). Logical depth can be summed up by the time taken to decompress an object described by a minimal algorithm. Logical depth is low when the algorithm has few computations to do. In physics, the question of existence of regularities in the world reduces to knowing if the world is algorithmically compressible (Davies, 1989; Wolfram, 2002). It is hence reasonable to ask whether our mental model of the world is itself an algorithmic compression.

In conclusion, Kolmogorov and logical depth are two complementary ways of understanding the complexity of objects. Theoretically, the length of a rule and its decompression time are respectively estimates of the Kolmogorov and the logical depth of a concept.

References

Anthony, M., & Biggs, N. (1992). *Computational learning theory*. Cambridge, MA: Cambridge University Press.

Arnauld, A., & Nicole, P. (1996). *Logic or the art of thinking*. Cambridge, UK: Cambridge university press (Original work published 1662).

Ashby, F. G., Alfonso-Reese, L. A., Turken, A. U., & Waldron, E. M. (1998). A neuropsychological theory of multiple systems in category learning. *Psychological Review, 105*, 442–481.

Ashby, F. G., Boyton, G., & Lee, W. W. (1994). Categorization response time with multidimensional stimuli. *Perception and Psychophysics, 55*, 11–27.

Axelrod, R. (1997). *The complexity of cooperation: Agent-based models of competition and collaboration*. Princeton, NJ: Princeton University Press.

Bennett, C. H. (1986). On the nature and origin of complexity in discrete, homogeneous, locally-interacting systems. *Foundations of Physics, 16*(6), 585–592.

Bourne, L. E. Jr. (1970). Knowing and using concepts. *Psychological Review, 77*, 546–556.

Bruner, J. S., Goodnow, J. J., & Austin, G. A. (1956). *A study of thinking*. New York: Wiley.

Bryant, R. E. (1986). Graph-based algorithms for Boolean function manipulation. *IEEE Transactions on Compilers, 35*(8), 677–691.

Chaitin, G. J. (1974). Information-theoretic computational complexity. *IEEE Transactions on Information Theory, 20*(1), 1015.

Chaitin, G. J. (1987). *Information, randomness and incompleteness: papers on algorithmic information theory*. New York: World Scientific.

Chaitin, G. J. (1990). *Algorithmic information theory*. Cambridge, MA: Cambridge University Press.

Chater, N., Vitányi, P. (2003). Simplicity: A unifying principle in cognitive science? *Trends in Cognitive Science, 7*(1), 19–22.

Davies, P. C. W. (1989). Why is the physical world so comprehensible? In W. H. Zurek (Ed.), *Complexity, entropy, and the physics of information*, pp. 61–70. New York: Addison-Wesley.

De Raedt, L. (1997). Logical settings for concept-learning. *Artificial Intelligence, 95*, 187–201.

Delahaye, J. P. (1993). Complexités. In J. P. Delahaye (Ed.), *Logique, informatique et paradoxe*. Paris: Pour la science.

Delahaye, J. P. (1994). *Information, complexité et hasard*. Paris: Hermès.

Duda, R. O., Hart, E., & Stork, D. G. (2001). *Pattern classification* (2nd ed.). New York: Wiley.

Estes, W. K. (1994). *Classification and cognition*. Oxford: Oxford University Press.

Fahlman, S. E., & Lebiere, C. (1990). The cascade-correlation learning architecture. *Technical report. CMU-CS-90–100*. Canergy-Mellon University.

Fass, D., & Feldman, J. (2002). Categorization under complexity: A unified MDL account of human learning of regular and irregular categories. In S. Becker, S. Thrun, & K. Obermayer (Eds.), *Advances in neural information processing systems, Proceedings of the 2002 conference* (Vol. 15). Cambridge, MA: MIT Press.

Feldman, J. (2000). Minimization of Boolean complexity in human concept learning. *Nature, 407*, 630–633.

Feldman, J. (2003a). A catalog of Boolean concepts. *Journal of Mathematical Psychology, 47*, 75–89.

Feldman, J. (2003b). The simplicity principle in human concept learning. *Cognition, 93*, 199–224.

Feldman, J. (2004). How surprising is a simple pattern? Quantifying "Eureka!". *Current Directions in Psychological Science, 12*(6), 227–232.

Ferber, J. (1999). *Multi-agent systems, an introduction to distributed artificial intelligence*. Harlow: Addison-Wesley.

Gell-Mann, M. (1994). *The quark and the jaguar: Adventures in the simple and the complex*. New York: Freeman and Company.

Gluck, M. A., & Bower, G. H. (1988a). Evaluating an adaptive network model of human learning. *Journal of Memory and Language, 27*, 166–195.

Gluck, M. A., & Bower, G. H. (1988b). From conditioning to category learning: an adaptive network model. *Journal of Experimental Psychology: General, 117*, 227–247.

Gold, E. M. (1967). Language identification in the limit. *Information and Control, 10*, 447–474.

Gosselin, F., & Schyns, P. G. (2001). Why do we SLIP to the basic level? Computational constraints and their implementation. *Psychological Review, 108*(4), 735–758.

Hahn, U., & Chater, N. (1997). Concepts and similarity. In K. Lamberts & D. Shanks (Eds.), *Knowledge, concepts and categories* (pp. 43–92). Cambridge, MA: MIT Press.

Hanson, S. J., Drastal, G. A., & Rivest, R. L. (1994a). *Computational learning theory and natural learning systems* (Vol. 1). Cambridge, MA: MIT Press.

Hanson, S. J., Petsche, T., Kearns, M., & Rivest, R. L. (1994b). *Computational learning theory and natural learning systems* (Vol. 2). Cambridge, MA: MIT Press.

Hovland, C. L. (1966). A communication analysis of concept learning. *Psychological Review, 59*, 461–472.

Hromkovič, J. (1997). *Communication complexity and parallel computing*. New York: Springer.

Huber, L. (2001). Visual categorization in pigeons. In R. G. Cook (Ed.), *Avian visual cognition* [On-line]. Available: www.pigeon.psy.tufts.edu/avc/huber/.

Hunt, E. B., Marin, J., & Stone, P. J. (1966). *Experiments in induction*. New York: Academic.

Huth, M., & Ryan, M. (2000). *Logic in computer science: Modeling and reasoning about systems*. London: Cambridge University Press.

Keanrs, M. J., Vazirani, U. V. (1994). *An introduction to computational learning theory*. Cambridge, MA: MIT Press.

Kolmogorov, A. N. (1965). Three approaches for defining the concept of information quantity. *Information Transmission, 1*, 3–11.

Kruschke, J. K. (1992). ALCOVE: An exemplar-based connectionist model of category learning. *Psychological Review, 99*, 22–44.

Lamberts, K. (1997). Process models of categorization. In K. Lamberts & D. Shanks (Eds.), *Knowledge, concepts and categories* (pp. 43–92). Cambridge, MA: MIT Press.

Lamberts, K. (2000). Information-accumulation theory of speeded categorization. *Psychological Review, 107*, 227–260.

Levine, M. (1966). Hypothesis behavior by humans during discrimination learning. *Journal of Experimental Psychology, 71*, 331–338.

Li, M., & Vitányi, P. (1997). *An introduction to Kolmogorov complexity and its applications*. New York: Springer Verlag.

Lockhead, G., & Pomerantz, J. R. (Eds.). (1991). *The perception of structure*. Washington: American Psychological Association.

Love, B. C., & Markman, A. B. (2003). The nonindependence of stimulus properties in human concept learning. *Memory and Cognition, 31*(5), 790–799.

Love, B. C., Medin, D. L., & Gureckis, T. M. (2004a). SUSTAIN: A network model of category learning. *Psychological Review, 111*, 309–332.

Love, B. C., Medin, D. L., & Gureckis, T. M. (2004b). SUSTAIN: A network model of category learning. *Psychological Review, 111*(2), 309–332.

Luce, R. D. (1963). Detection and recognition. In R. D. Luce, R. R. Bush, E. Galanter (Eds.), *Handbook of mathematical psychology* (pp. 103–189). New York: Wiley.

Mathy, F. (2002). L'apprenabilité des concepts évaluée au moyen d'un modèle multi-agent de la complexité des communications en mémoire de travail. Unpublished Doctoral Dissertation. Université de Reims, France.

Mathy, F., Bradmetz, J. (1999). Un modèle multi-agent de la complexité conceptuelle. In M. P. Gleizes, P. Marcenac (Eds.), *Ingénierie des systèmes multi-agents: acte des 7e Journées Francophones d'Intelligence Artificielle et Systèmes Multi-Agents*. Paris: Hermès. (pp. 343–345).

Mathy, F., & Bradmetz, J. (2004). A theory of the graceful complexification of concepts and their learnability. *Current Psychology of Cognition, 22*(1), 41–82.

Medin, D. L., & Schaffer, M. M. (1978). A context theory of classification learning, *Psychological Review, 85*, 207–238.

Minsky, M. L. (1985). *The society of mind*. New York, NY: Simon and Schuster (French translation: *La société de l'esprit*. Paris: Inter Editions, 1988).

Mitchell, T. (1997). *Machine learning*. New York, NY: McGraw-Hill.

Myung, I. J. (1994). Maximum entropy interpretation of decision bound and context models of categorization. *Journal of Mathematical Psychology, 38*, 335–365.

Nosofsky, R. M. (1986). Attention, similarity, and the identification-categorization relationship. *Journal of Experimental Psychology: General, 115*, 39–57.

Nosofsky, R. M., & Alfonso-Reese, L. A. (1999). Effects of similarity and practice on speeded classification response times and accuracies: Further tests of an exemplar-retrieval model. *Memory and Cognition, 27*, 78–93.

Nosofsky, R. M., Gluck, M. A., Palmeri, T. J., McKinley, S. C., & Gauthier, P. (1994a). Comparing models of rules-based classification learning: A replication and extension of Shepard, Hovland, and Jenkins (1961). *Memory and Cognition, 22*, 352–369.

Nosofsky, R. M., Kruschke, J. K., & McKinley, S. C. (1992). Combining exemplar-based category representations and connectionist learning rules. *Journal of experimental Psychology: Learning, Memory, and Cognition, 18*, 211–233.

Nosofsky, R. M., Palmeri, T. J. (1997). An exemplar-based random walk model of speeded classification. *Psychological Review, 104*, 266–300.

Nosofsky, R. M., Palmeri, T. J., & McKinley, S. C. (1994b). Rule-plus-exception model of classification learning. *Psychological Review, 101*(1), 53–79.

Osherson, D. N., & Smith, E. E. (1981). On the adequacy of prototype theory as a theory of concepts. *Cognition, 9*, 35–58.

Osherson, D. Stob, M., & Weistein, S. (1986). *Systems that learn: an introduction to learning theory for cognitive and computer scientists*. Cambridge, MA: MIT Press.

Pothos, E. M., & Chater, N. (2002). A simplicity principle in unsupervised human categorization. *Cognitive Science, 26*, 303–343.

Quinlan, J. R. (1986). Induction of decision trees. *Machine Learning, 1*, 81–106.

Richards, W., & Bobick, A. (1988). Playing twenty questions with nature. In Z. W. Pylyshyn (Ed.), *Computational processes in human vision: an interdisciplinary perspective* (pp. 3–26). Norwood, NJ: Ablex.

Richards, W., McKay, B. D., & Richards, D. (2002). The probability of collective choice with shared knowledge structures. *Journal of Mathematical Psychology, 46*, 338–351.

Rips, L. J., Shoben, E. J., Smith, E. E. (1973). Semantic distance and the verification of semantic relations. *Journal of Verbal Learning and Verbal Behavior, 12*, 1–20.

Roberts, S., & Pashler, H. (2000). How persuasive is a good fit? A comment on theory testing. *Psychological Review, 107*, 358–367.

Rosch, E. (1973). Natural categories. *Cognitive Psychology*, 4, 328–350.

Rosch, E., & Mervis, C. B. (1975). Family resemblances: studies in the internal structure of categories. *Cognitive Psychology, 7*, 573–605.

Selfridge, O. (1959). Pandemonium: a paradigm for learning. In D. V. Blake, A. M. Uttley (Eds.), *Proceedings of the symposium on mechanisation of thought processes* (pp 511–529). London: H. M. Stationary Office.

Shavlik, J. W., & Dietterich, T. G. (Eds.). (1990). *Readings in machine learning*. San Mateo, CA: Morgan Kaufmann.

Shepard, R. N. (1987). Toward a universal law of generalization for psychological science. *Science, 237*, 1317–1323.

Shepard, R. N., Hovland, C. L., & Jenkins, H. M. (1961). Learning and memorization of classifications. *Psychological Monographs, 75* (13, whole No. 517).

Shin, H. J., & Nosofsky, R. M. (1992). Similarity-scaling studies of dot-pattern classification and recognition. *Journal of Experimental Psychology: General, 121*, 278–304.

Siegler, R. S. (1977). The twenty questions game as a form of problem solving. *Child Development, 48* (2), 395–403.

Smith, E. E., & Medin, D. L. (1981). *Categories and concepts*. Cambridge, MA: Harvard University Press.

Townsend, J. T., & Wenger, M. J. (2004). The serial-parallel dilemma: A case study in a linkage of theory and method. *Psychonomic Bulletin & Review, 11*, 391–418.

Valiant, L. G. (1984). A theory of the learnable. *Communication of the Association for Computing Machinery, 27*, 1134–1142.

Wolfram, S. (2002). *A new kind of science*. Champaign, IL: Wolfram Media.

Yamauchi, T., Love, B. C., & Markman, A. B. (2002). Learning nonlinearly separable categories by inference and classification. *Journal of Experimental Psychology: Learning, Memory and Cognition, 28*, 585–593.

Zucchini, W. (2000). An introduction to model selection. *Journal of Mathematica Psychology, 44*, 41–61.